KEY

Math 152 Quiz #1 Name:

Directions: Please show all your work to receive credit. You will be graded on the clarity and organization of your work as well as its accuracy.

1. (10 points) Use the definition of the definite integral to evaluate $\int_0^b x^3\, dx$, where b is a constant. You will need to use the fact that
$\Sigma_{i=1}^{n} i^3 = (n(n+1)/2)^2$.
Be precise in your notation to receive full credit.

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2. (7 points) Approximate $\int_1^9 \frac{x-3}{x+1}\, dx$ by evaluating a Riemann sum with four subintervals of equal widths and choosing the left endpoint of each subinterval for your sample points.

7

3. (8 points) Evaluate each integral by interpreting it in terms of signed area.

8

a. $\int_{-3}^{0}(1+\sqrt{9-x^2})\, dx$

b. $\int_{-2}^{2}(-1-\mid x \mid)\, dx$

$$1)\ \int_0^b x^3 dx = \lim_{n\to\infty}\left(\frac{b}{n}\right)\left(\left(\frac{b}{n}\right)^3+\left(\frac{2b}{n}\right)^3+\cdots+\left(\frac{nb}{n}\right)^3\right)$$

$*\Delta x = \frac{b}{n}$

$$= \lim_{n\to\infty}\frac{b^4}{n^4}\left(1^3+2^3+\cdots+n^3\right)$$

$\sum_{i=1}^{n} i^3$ (the fact)

$$= \lim_{n\to\infty}\left(\frac{b^4}{n^4}\right)\sum_{i=1}^{n} i^3 = \lim_{n\to\infty}\left(\frac{b^4}{n^4}\right)\left(\frac{n(n+1)}{2}\right)^2$$

$$= \lim_{n\to\infty}\frac{b^4\cdot \cancel{n^2}\cdot(n+1)^2}{\cancel{n^4}\cdot 4}$$

$$= \frac{b^4}{4}\lim_{n\to\infty}\left(1+\frac{2}{n}+\frac{1}{n^2}\right)$$

$$= \frac{b^4}{4}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n}\right) = \frac{b^4}{4}$$

$$2.\ \int_1^9 \frac{x-3}{x+1} = \sum_{i=1}^{4} f(x_{i-1})\,dx \approx 2\left[f(1)+f(3)+f(5)+f(7)\right]$$

$$\Delta x = \frac{9-1}{4} = \frac{8}{4} = 2$$

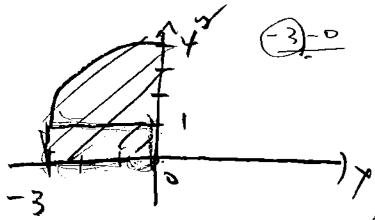
$$= 2\left[-\frac{2}{2}+\frac{0}{4}+\frac{2}{6}+\frac{4}{8}\right]$$

$$= 2\left[-1+0+\frac{1}{3}+\frac{1}{2}\right]$$

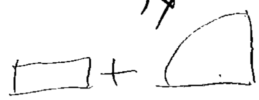
$$= 2\left(\frac{-6+2+3}{6}\right) = \frac{-\cancel{2}}{\cancel{6}_3} = -\frac{1}{3}$$

3. a) $\int_{-3}^{0} (1+\sqrt{9-x^2})\,dx = (1\times 3) + (3\times 3\times \pi \times \frac{1}{4})$

$= 3 + \frac{9\pi}{4}$



$-3-0$



b) $\int_{-2}^{2} (-1-|x|)\,dx = -\left(\frac{1}{\cancel{2}}(1+3)\cancel{2}\right) - \left(\frac{1}{\cancel{2}}(1+3)\cancel{2}\right)$

$= -4-4 = -8$