

**Directions:** Please show all your work to receive credit. You will be graded on the clarity and organization of your work as well as its accuracy. Simplify all answers as fully as possible. Evaluate all trigonometric functions of common values. Do all your work on a separate piece of paper.

1. (6 points) Evaluate  $\int_1^4 \frac{6x+1}{2x-10} dx$ .
2. (6 points) Find  $\int \sinh \sqrt{x} dx$
3. (8 points) The region bounded by the curve  $y = f(x)$  and the  $x$ -axis is rotated about the  $y$ -axis to form a solid. The only information known about the function  $y = f(x)$  is that the points  $(0, 0)$ ,  $(4, 6)$ ,  $(5, 12)$ ,  $(10, 6)$ , and  $(11, 0)$  lie on its graph. Use Simpson's rule and these data points to approximate the volume of the solid.
4. (4 points) The speed of a car is 40 mph at 1:00pm, 50 mph at 1:15pm, and 30 mph at 1:30pm. Use the trapezoid rule to estimate the average speed of the car between 1:00pm and 1:30pm.
5. (8 points) Find the area of the region bounded by the smaller loop of the limaçon with polar equation  $r = 1 + 2 \sin \theta$ .
6. (8 points) A small pebble is caught in the spokes 5 inches from the center of a bicycle wheel having a radius of 13 inches. The pebble is initially at its high point, 18 inches above the ground. Set up, but **do not evaluate**, a definite integral that gives the distance traveled by the pebble, if the wheel then turns through an angle of 90 degrees.

1. 
$$\int_1^4 \frac{6x+1}{2x-10} dx = \overset{\text{KEY}}{\int_1^4 \left( 3 + \frac{31}{2x-10} \right) dx}$$

$$= \left( 3x + \frac{31}{2} \ln |2x-10| \right) \Big|_{x=1}^{x=4}$$

$$= 9 + \frac{31}{2} \ln \frac{1}{4}$$

2. Let  $u = \sqrt{x}$   
 then  $u^2 = x$  and  $\frac{dx}{du} = 2u$ , so  $dx = 2u du$ ,  
 and

$$\int \sinh \sqrt{x} dx = 2 \int u \sinh u du$$

$$= 2 (u \cosh u - \sinh u)$$

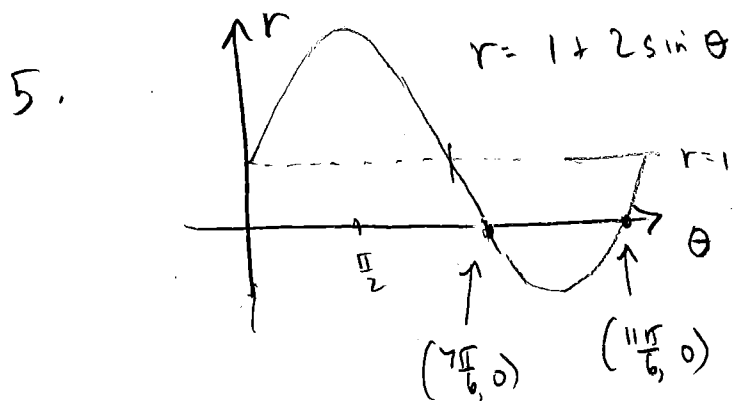
$$= 2(\sqrt{x} \cosh \sqrt{x} - \sinh \sqrt{x})$$

3. 
$$V = \int_0^{12} A(y) dy \approx \frac{6}{3} (A(0) + 4A(6) + A(12))$$

$$= 2\pi (11^2 + 4(10^2 - 4^2) + 0)$$

$$= 914\pi \text{ units}^3$$

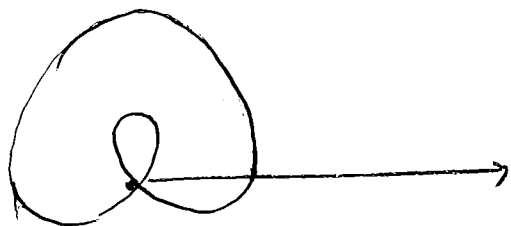
4. Avg speed  $\sim \frac{1}{4} (40 + 2(50) + 30)$   
 $= 42.5 \text{ mph}$



$$1 + 2 \sin \theta = 0 \Rightarrow$$

$$\sin \theta = -\frac{1}{2} \Rightarrow$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6} \text{ if } 0 \leq \theta \leq 2\pi$$

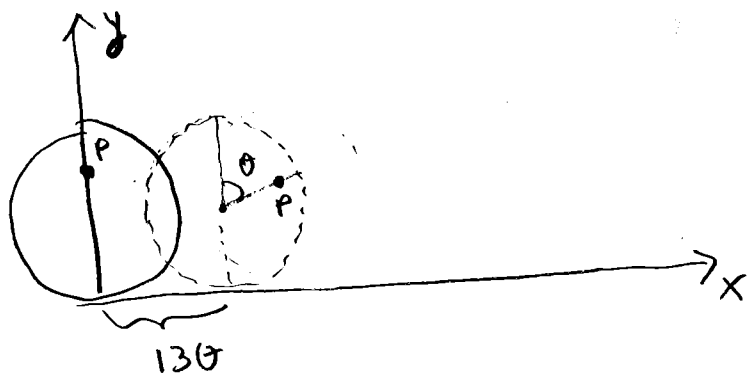


$$A = \frac{1}{2} \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} (1 + 2 \sin \theta)^2 d\theta$$

$$= \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} (1 + 4 \sin \theta + 4 \sin^2 \theta) d\theta$$

$$= \pi - \frac{3\sqrt{3}}{2} \text{ units}^2$$

6. Let  $\theta$  denote the angle through which the wheel has turned.



Then  $x = 13\theta + 5 \sin \theta$   
 $y = 13 + 5 \cos \theta$

The distance traveled by P for  $0 \leq \theta \leq \pi/2$  is

$$\int_0^{\pi/2} \sqrt{(13 + 5 \cos \theta)^2 + 25 \sin^2 \theta} d\theta$$