Math 152 Test #1 Name: KEY

Directions: Please show all your work to receive credit. You will be graded on the clarity and organization of your work as well as its accuracy. Simplify all answers as fully as possible. Do all your work on a separate piece of paper.

1. (3 points) Complete this version of the fundamental theorem. Use words, not symbols:

   Integrating the rate of change of a (differentiable) function over an interval gives ...

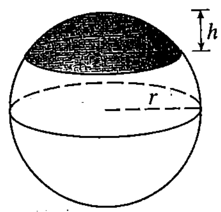
2. (10 points) Evaluate each of the following integrals.

a. $\int \frac{e^{5x}}{(1+e^{5x})^2}\,dx$

b. $\int_0^1 \frac{10}{12-6x}\,dx$

3. (5 points) If the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = a$ into two regions of equal area, express b in terms of a. Assume $a > 0$.

4. (8 points) Use integration to find the volume of the cap of a sphere with radius r and height h.

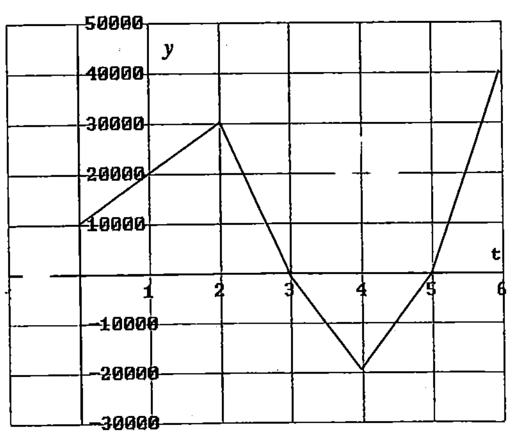


5. (8 points) The base of a solid is the region bounded by the curves $y = \frac{1}{9}x$ and $y = \sqrt[3]{x}$ in the first quadrant. Cross-sections perpendicular to the y-axis are squares. Set up, **but do not evaluate**, a definite integral that gives the volume of the solid.

6. (6 points) The function $r(t)$ gives the growth rate (in people/year) of a population of a city t years after the year 2000. Define $A(t) = \int_2^t r(x)\,dx$. Use the graph of $y = r(t)$ shown below to answer the following questions.

a. Give the exact value of $A(4)$. Explain its meaning. Include units.

b. Give the exact value of $A'(4)$. Explain its meaning. Include units. If it is undefined, explain why.



$y = r(t)$

1. Integrating the rate of change of a function over an interval gives the change in the function over the interval.

2 a) $\displaystyle\int \frac{e^{5x}}{(1+e^{5x})^2}\,dx. = \frac{1}{5}\int \frac{du}{u^2} = -\frac{1}{5}u^{-1} + C$

$\displaystyle = -\frac{1}{5}\cdot\frac{1}{(e^{5x}+1)} + C$

Let $u = e^{5x}+1$

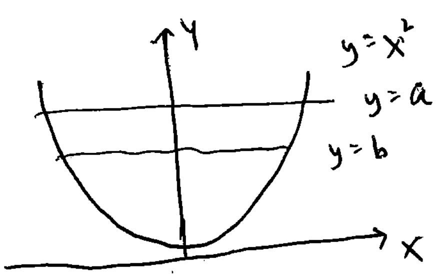
Then $du = 5e^{5x}dx$

and $\frac{1}{5}du = e^{5x}dx$

b) $\displaystyle\int_0^1 \frac{10}{12-6x}\,dx = -\frac{10}{6}\int_{12}^{6} \frac{du}{u} = -\frac{10}{6}\ln u\Big]_{12}^{6} = \frac{5}{3}\ln 2$

Let $u = 12-6x$

Then $dx = -\frac{1}{6}du$

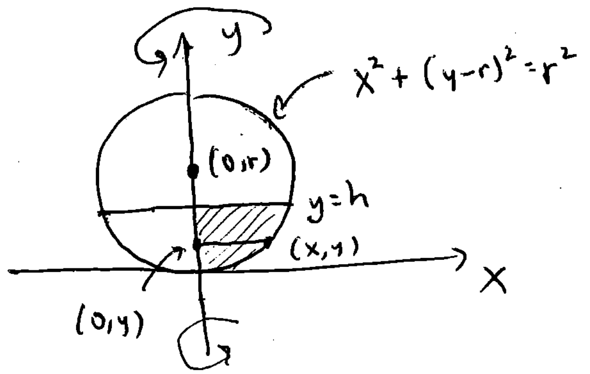
3. 

$\displaystyle 2\int_0^b \sqrt{y}\,dy = \int_0^a \sqrt{y}\,dy \Rightarrow$

$\displaystyle 2\cdot\frac{2}{3}y^{3/2}\Big]_0^b = \frac{2}{3}y^{3/2}\Big]_0^a \Rightarrow$

$\frac{4}{3}b^{3/2} = \frac{2}{3}a^{3/2} \Rightarrow$

$b = a/\sqrt[3]{4}.$

4. 

$A = \pi x^2$

$= \pi\left(r^2-(y-r)^2\right)$

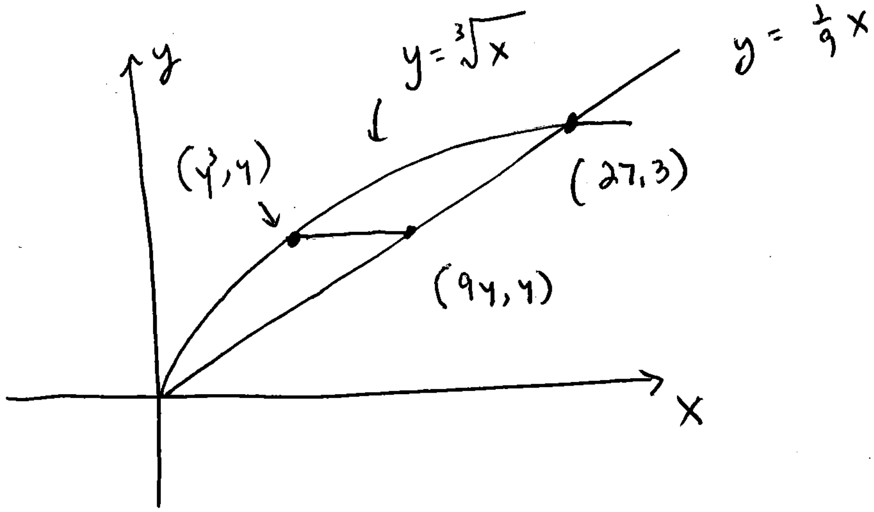
$\displaystyle V = \int_0^h A(y)\,dy$

$\displaystyle = \pi\int_0^h (2yr-y^2)\,dy$

$\displaystyle = \pi\left(ry^2-\frac{1}{3}y^3\right)\Big]_{y=0}^{y=h}$

$= \pi\left(rh^2-\frac{1}{3}h^3\right)$

5.



$y = \frac{1}{9}x \Rightarrow x = 9y$

$y = \sqrt[3]{x} \Rightarrow x = y^3$

The curves intersect when $9y = y^3$ or

$y(y^2 - 9) = 0$

$y = 0, 3, -3$

The volume of the solid is $\int_0^3 (9y - y^3)^2 \, dy$.

6. a) $A(4) = 5{,}000$ people. The population increased by 5,000 people between 2002 and 2004.

b) $A'(4) = -20{,}000$ people/yr.

The population was decreasing at the rate of 20,000 people/yr in 2004.