KEY

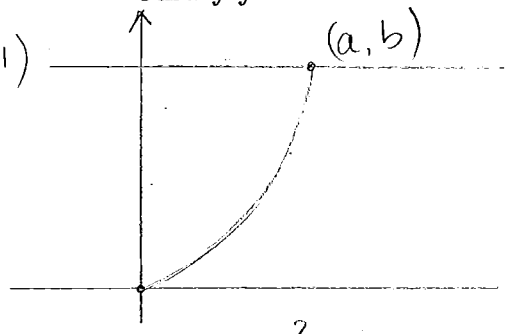
Math 152 Quiz #8 Name: _____

Directions: Please show all your work and use correct mathematical notation to receive credit. Do all your work **below** the last problem.

1. (20 points) Let a and b be positive constants. Find the coordinates of the centroid of the region in the first quadrant bounded by the y-axis, the line $y = b$, and the parabola with vertex at the origin that passes through the point (a, b).

2. (5 points) Is the function $y = 5e^x - x - 1$ a solution to the differential equation $\frac{dy}{dx} = x + y$? Justify your answer.

1)



$y = cx^2$

$b = c \cdot a^2$

$c = \frac{b}{a^2}$ so $y = \frac{b}{a^2} x^2$

$$x_{cm} = \frac{\int_0^a x\, h(x)\, dx}{\int_0^a h(x)\, dx} = \frac{\int_0^a x\left(b - \frac{b}{a^2}x^2\right) dx}{\int_0^a \left(b - \frac{b}{a^2}x^2\right) dx}$$

$$= \frac{b\frac{x^2}{2} - \frac{b}{a^2}\cdot\frac{1}{4}x^4\Big]_0^a}{bx - \frac{b}{a^2}\frac{x^3}{3}\Big]_0^a} = \frac{\frac{a^2 b}{2} - \frac{a^2 b}{4}}{ba - \frac{ba}{3}}$$

$$= \frac{\frac{a^2 b}{4}}{\frac{2}{3}ba} = \frac{3}{8}a$$

$$y_{cm} = \frac{\int_0^b y\, w(y)\, dy}{\int_0^b w(y)\, dy} = \frac{\int_0^b y\left(\frac{a}{\sqrt{b}}\cdot\sqrt{y}\right) dy}{\int_0^b \left(\frac{a}{\sqrt{b}}\sqrt{y}\right) dy}$$

$$= \frac{\frac{a}{\sqrt{b}}\int_0^b y^{\frac{3}{2}}\, dy}{\frac{a}{\sqrt{b}}\int_0^b y^{\frac{1}{2}}\, dy} = \frac{\frac{2}{5}y^{\frac{5}{2}}\Big]_0^b}{\frac{2}{3}y^{\frac{3}{2}}\Big]_0^b}$$

$$= \frac{3}{5}\cdot\frac{b^2\sqrt{b}}{b\sqrt{b}} = \frac{3}{5}b$$

So the centroid of the region is $\left(\frac{3}{8}a, \frac{3}{5}b\right)$

2. If $y = 5e^x - x - 1$,

then $\frac{dy}{dx} = 5e^x - 1$.

Also, since $y = 5e^x - x - 1$,

$$y + x = 5e^x - 1.$$

thus,

$$\frac{dy}{dx} = y + x,$$

and $y = 5e^x - x - 1$

is a soln. to

$$\frac{dy}{dx} = y + x.$$