Math 151 Implicit Differentiation

1. Let $\mathcal{E}$ be the ellipse passing through $P(3,0)$ with focus $(0,0)$ and directrix $y = -6$. Find an equation of the tangent line to $\mathcal{E}$ at P.

2. Let $\mathcal{C}$ be the set of points twice as far from $(0,0)$ as from $(9,4)$.
a. Verify that $P(6,8)$ lies on $\mathcal{C}$.
b. Find an equation of the tangent line to $\mathcal{C}$ at P.

3. Let $\mathcal{P}$ be the set of points equidistant from the point $(2,0)$ and the line $x = -2$.
a. Verify that $Q(8,8)$ lies on $\mathcal{P}$.
b. Find an equation of the tangent line to $\mathcal{P}$ at Q.

4. Let a be a positive constant. Prove that the segments of the tangent lines cut off by the coordinate axes to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ have constant length.

5. A diameter of an ellipse is a segment passing through its center whose endpoints lie on the ellipse. Let $\overline{AB}$ and $\overline{CD}$ be diameters of an ellipse. Prove that if the tangents to the ellipse at A and B are parallel to CD, then the tangents at C and D are parallel to AB. You may assume that an equation of an ellipse centered at the origin is of the form $(x/a)^2 + (y/b)^2 = 1$.

6. A colony of bacteria triples every 4 hours. Find the population when the population is growing at the rate of 10,000 bacteria/hour.

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