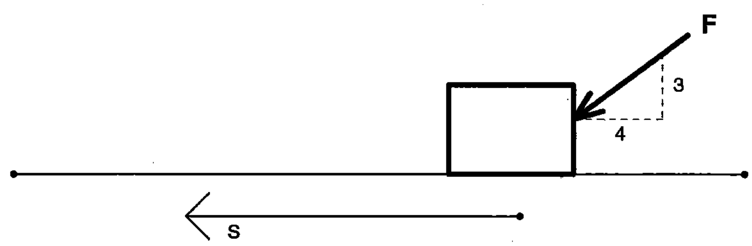
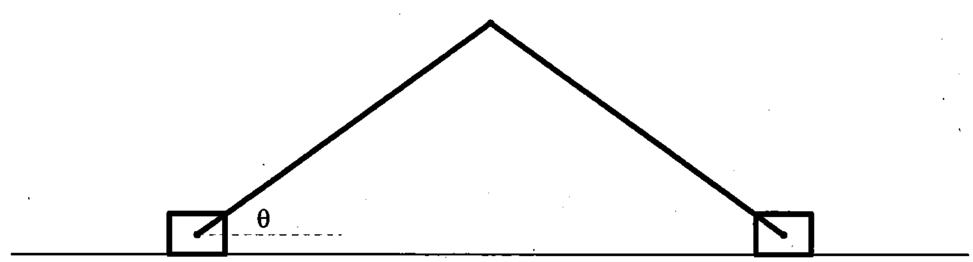
Engineering 230 Test 3 Name: ______________________

Directions: Show work and be neat and organized to receive credit. Give brief explanations of your work. Define all variables not already given in the statement of the problem.

1. (20 points) A 64-lb. block is subjected to a force having a constant direction and a magnitude $F = 4s$, where F is measured in pounds and s is the (leftward) displacement of the block measured in feet. When $s = 0$ feet the block is moving to the left with a speed of 16 ft/s. If the coefficient of kinetic friction between the block and the surface is $\mu_k = 0.5$, determine the displacement of the block when it stops. Use $g = 32\ ft/s^2$.

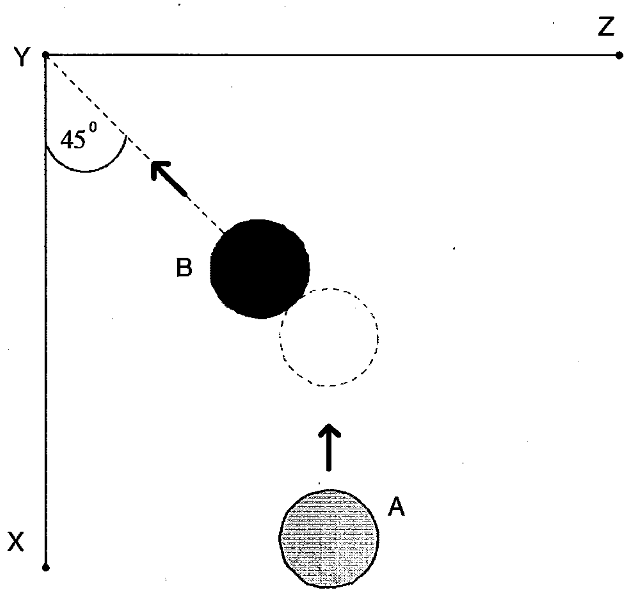


2. (20 points) The system below consists of two bars of length L meters which are pin connected to each other and to two small blocks. Each bar and each block has a mass of m kg. If the bars are released from rest from $\theta = 90°$, determine their angular velocities when $\theta = 30°$. Assume that the blocks slide on a smooth surface.



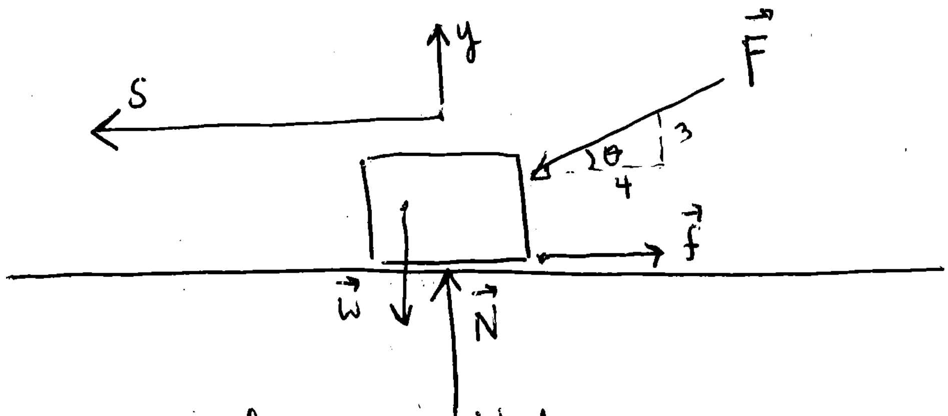
3. (20 points) A projectile is fired from ground level with an initial speed of v_0 m/s at an angle of 60° above the horizontal. When it reaches its highest point it explodes into two fragments of equal mass. If one of the fragments travels vertically upward with a speed of v_0 m/s immediately after the explosion, determine the speed of the other fragment as it hits the ground. Ignore the impulse of gravity during the explosion.

4. (20 points) Two smooth billiard balls A and B have the same mass. If A strikes B with a speed of v_A m/s as shown, determine the speed of ball A just after the collision. Ball B is initially at rest. Before the collision ball A is moving parallel to side XY of the table and after the collision ball B heads directly to the corner pocket Y. The coefficient of restitution of the collision is e.



Test 3 - Solutions

1.



Let m be the mass of the block
Let $\vec{N}$ be the normal force of the surface on the block.
Let $\vec{w}$ be the weight of the block.
Let $\vec{f}$ be the frictional force of the surface on the block.
Let S be the position of the block when it stops

Then $\sum F_y = 0 \Rightarrow N = mg \sin\theta$

$$N = mg + F\sin\theta = 64 + (4s)\left(\tfrac{3}{5}\right) = 64 + \tfrac{12}{5}s$$

$$\sum F_x = \tfrac{16}{5}s - \tfrac{1}{2}\left(64 + \tfrac{12}{5}s\right) = 2s - 32$$

$$\int_0^S (2s-32)\,ds = 0 - 16^2$$

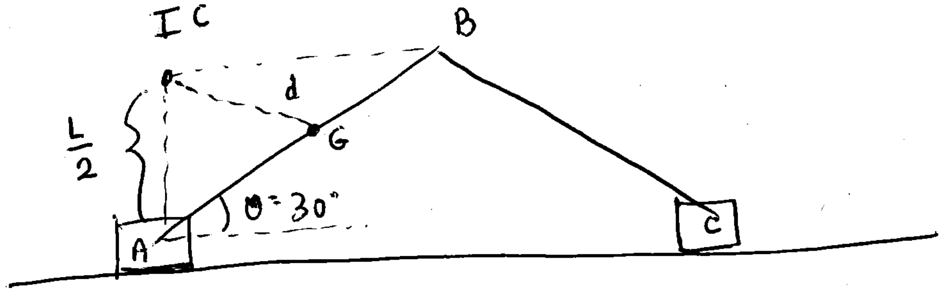
$$S^2 - 32S + 256 = 0$$

$$(S-16)^2 = 0$$

$$S = 16$$

The block stops at $S = 16$ feet.

2.



The work done by gravity on the two bars between $\theta = 90°$ and $\theta = 30°$ is

$$U_g = 2mg\left(\frac{L}{2} - \frac{L}{2}\sin 30°\right) = \frac{1}{2}mgL$$

Let ω be the angular speed of each bar and let I_{IC} be the moment of inertia of each bar about its instantaneous center of rotation. Let v denote the speed of each block. Then

$$V = (L\sin 30°)\omega = \frac{L}{2}\omega \quad \text{and}$$

$$I_{IC} = \frac{1}{12}mL^2 + md^2 = \frac{1}{12}mL^2 + m\left(\frac{1}{2}L\right)^2 = \frac{1}{3}mL^2.$$

By the work-kinetic energy theorem,

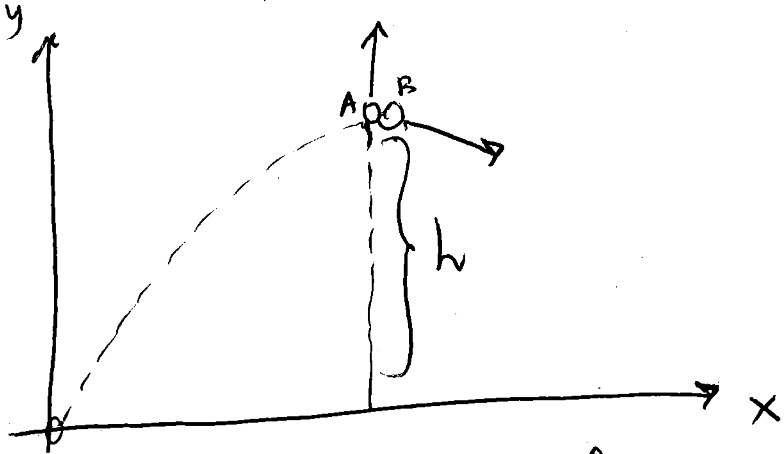
$$\frac{1}{2}mgL = 2\cdot\frac{1}{2}mv^2 + 2\cdot\frac{1}{2}\cdot I_{IC}\,\omega^2, \quad \text{or}$$

$$\frac{1}{2}mgL = m\left(\frac{1}{2}L\omega\right)^2 + \frac{1}{3}mL^2\omega^2.$$

Solving gives

$$\omega = \sqrt{\frac{6}{7}g/L} \text{ rad/s}$$ as the angular speed of each bar. Bar AB rotates clockwise, bar BC rotates counterclockwise.

3.



Let $M = 2m$ be the mass of the original projectile.

Let h be its maximum height.

Then $$\frac{1}{2} M v_0^2 = \frac{1}{2} M \left[V_0(\cos 60^\circ)\right]^2 + Mgh,$$ so

$$gh = \frac{3}{8} v_0^2.$$

Let A denote the piece that goes straight up and B denote the second piece (after the explosion). Let $\vec{V}_B$ denote the velocity of B immediately after the explosion. Since momentum is conserved during the explosion,

$$m_A v_0 + m_B (V_B)_y = 0 \Rightarrow (V_B)_y = -V_0$$

and $$m_A (0) + m_B (V_B)_x = 2m(V_0 \cos 60^\circ) \Rightarrow (V_B)_x = V_0$$

(Note $m_A = m_B = m$). So $V_B^2 = 2V_0^2$.

Let $\vec{V}_B^f$ denote the velocity of B as it hits the ground.

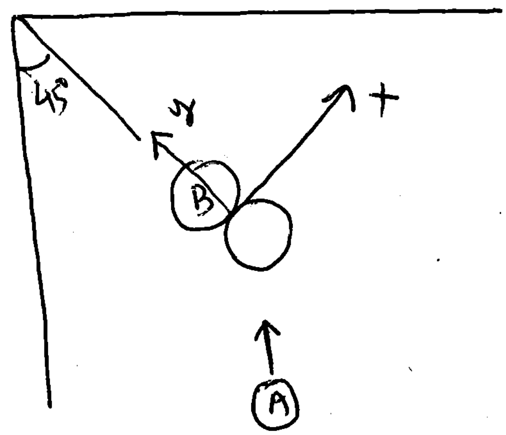
Then $$\frac{1}{2} m v_B^2 + mgh = \frac{1}{2} m (V_B^f)^2 \Rightarrow$$

$$\frac{1}{2}(2V_0^2) + \frac{3}{8} V_0^2 = \frac{1}{2}(V_B^f)^2 \Rightarrow (V_B^f)^2 = \frac{11}{4} V_0^2.$$

Thus, the speed of fragment B when it hits the ground is

$\frac{\sqrt{11}}{2} V_0$ m/s.

4.



Let m denote the mass of each ball.

Establish the above coordinate system. Let $\vec{V}_A$ denote the velocity of A before the collision and let $\vec{V}_B$ denote the velocity of B after the collision. Also let $\vec{V}_A^f$ denote the velocity of A after the collision. Since there are no impulsive forces acting in the x-direction,

$$(V_A^f)_x = (V_A)_x = V_A \cdot \frac{\sqrt{2}}{2}.$$

Since momentum is conserved (in the y-direction),

$$(1) \quad m \cdot \frac{\sqrt{2}}{2} V_A = m V_B + m (V_A^f)_y .$$

By definition,

$$(2) \quad e = \frac{V_B - (V_A^f)_y}{\frac{\sqrt{2}}{2} V_A - 0} \Rightarrow (V_A^f)_y + e \cdot \frac{\sqrt{2}}{2} V_A = V_B$$

Substituting (2) into (1) gives

$$\frac{\sqrt{2}}{2} V_A = (V_A^f)_y + e \frac{\sqrt{2}}{2} V_A + (V_A^f)_y \Rightarrow (V_A^f)_y = \frac{\sqrt{2}}{4}(1-e)V_A$$

Thus, $V_A^f = \sqrt{(V_A^f)_x^2 + (V_A^f)_y^2} = V_A \sqrt{\frac{1}{2} + \frac{1}{8}(1-e)^2}$.

The speed of A after the collision is $V_A \sqrt{\frac{1}{2} + \frac{1}{8}(1-e)^2}$ m/s.