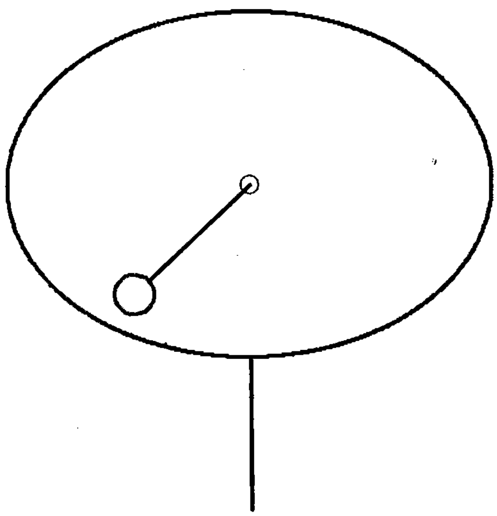
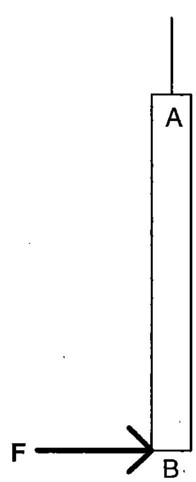
Engineering 230 Final Name: ____________________

Directions: Show work and be neat and organized to receive credit. Give brief explanations of your work. Define all variables not already given in the statement of the problem.

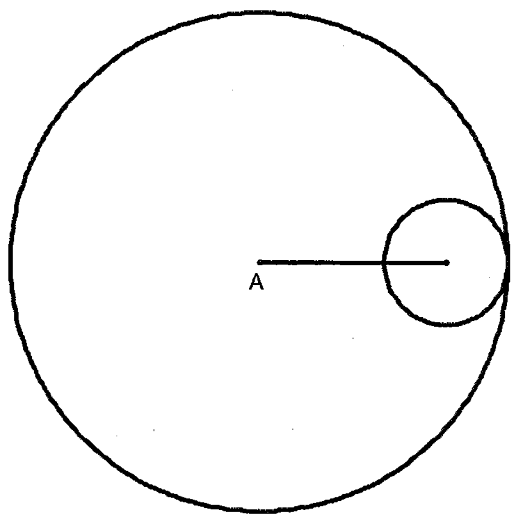
1. (20 points) A ball is moving in a circle of radius 2 feet with a speed of 4 ft/s on a horizontal table. If the attached cord is then pulled down through the hole with a constant speed of 3 ft/s, determine the time required for the ball to reach a speed of 8 ft/s. Neglect friction and the size of the ball.



2. (20 points) A slender rod with mass m kg and length L meters is suspended at its top end A by a cord attached to the ceiling. If a horizontal force of magnitude F newtons is applied to the rod at its bottom end B, determine the magnitude and direction of the accleration of the top end A when the force is first applied. The rod is initially at rest.



3. (20 points) One end of a uniform bar of mass m kg and length $3r$ meters is pin supported at A. The other end is pin-attached to a uniform disk of mass $2m$ kg and radius r meters. The disk rolls without slipping on the inside of a fixed circular track of radius $4r$ meters when the system is released from rest in the horizontal position. Determine the angular velocity of the rod when it has turned through an angle of θ.



4. (20 points) A hoop of radius r is thrown horizontally onto a rough horizontal surface such that the center G of the hoop has an initial speed of v_G. Determine the amount of backspin, ω_0 (measured in rad/s), the hoop must be given so that it stops spinning at the same instant the center stops moving.

5. (20 points) A solid uniform ball of mass m and radius r rolls down an inclined plane for which the coefficient of static friction is μ. If the ball is released from rest, determine the maximum angle of the incline such that the ball rolls without slipping.

1. Let V_θ be the transverse component of the velocity of the ball when its speed is 8 ft/s. Then

$$8 = \sqrt{3^2 + V_\theta^2}, \quad \text{so}$$

$$V_\theta = \sqrt{55}.$$

Momentum about the center of the table is conserved (since the force of tension is directed toward the center). So if r is the distance from the ball to the center when its speed is 8 ft/s,

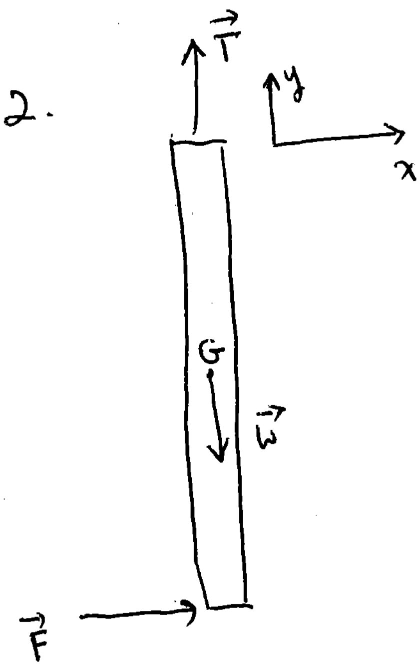
$$\sqrt{55} \cdot r = 2(4), \quad \text{so}$$

$$r = 8/\sqrt{55} \sim 1.08 \text{ ft}.$$

The time required for the ball to attain a speed of 8 ft/s is

$$\frac{(2 - 1.08)\text{ft}}{3 \text{ ft/s}} \sim .307 \text{ sec}.$$

2.



Let $\vec{T}$ be the tension in the rope at the moment the bar is struck, and let α be the bar's angular acceleration when it is struck. Then

$$\sum M_G = I_G \alpha \Rightarrow$$

$$F \cdot \frac{L}{2} = \frac{1}{12} m L^2 \alpha \Rightarrow$$

$$\alpha = \frac{6F}{mL}.$$

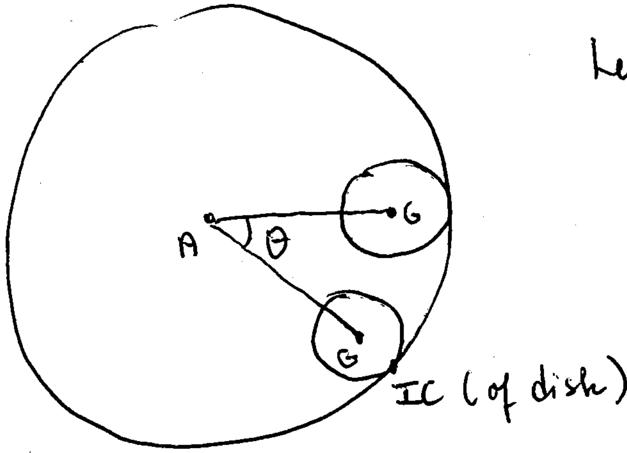
Since $(a_G)_y = 0$ and $(a_G)_x = \frac{F}{m}$,

$$\vec{a}_A = \vec{a}_G + \vec{a}_{A/G}$$

$$= \left(\frac{F}{m}\right)\vec{i} - \left(\frac{L}{2} \cdot \alpha\right)\vec{i} = \left(\frac{F}{m} - \frac{3F}{m}\right)\vec{i} = \frac{-2F}{m}\vec{i}$$

The acceleration of the bar has magnitude $2F/m$ m/s^2 in the direction opposite $\vec{F}$.

3.



Let ω be the angular velocity of the rod after it has turned through θ radians.

The work done by gravity on the system is

$$U_g = mg \cdot \frac{3r}{2}\sin\theta + 2mg \cdot 3r\sin\theta = \frac{15}{2} mgr\sin\theta$$

The speed, V_G, of G after the bar has rotated θ radians is

$$V_G = (3r)\omega = r\,\omega_{disk}, \quad \text{so} \quad \omega_{disk} = 3\omega.$$

Then $T_{bar} = \frac{1}{2}\left(\frac{1}{3}m(3r)^2\right)\omega^2 = \frac{3}{2}mr^2\omega^2$ and

$$T_{disk} = \frac{1}{2} I_{disk}\,\omega_{disk}^2 = \frac{1}{2}\left(\frac{3}{2}\cdot 2mr^2\right)(3\omega)^2 = \frac{27}{2}mr^2\omega^2,$$

where I_{disk} is the moment of inertia of the disk about its I.C.

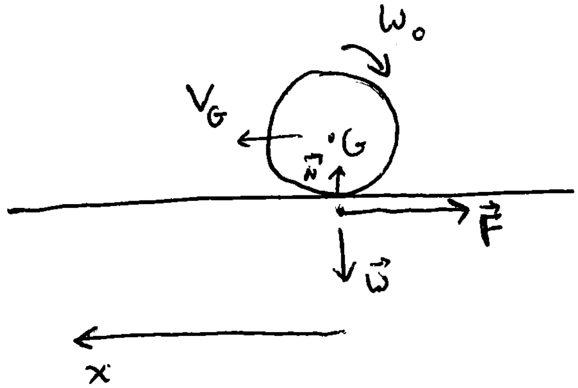
Thus,

$$\frac{15}{2}mgr\sin\theta = \frac{3}{2}mr^2\omega^2 + \frac{27}{2}mr^2\omega^2, \quad \text{so}$$

$$\omega = \sqrt{\frac{1}{2}\frac{g}{r}\sin\theta} \text{ rad/s.}$$

The angular velocity of the bar is $\sqrt{\frac{1}{2}\frac{g}{r}\sin\theta}$ rad/s clockwise.

4.



Let $\vec{F}$ be the frictional force of the ground on the wheel and let α be the magnitude of the wheel's angular acceleration.

$$\sum M_G = I_G \alpha \quad \Rightarrow$$

$$Fr = mr^2\alpha \quad \Rightarrow$$

$$\alpha = \frac{F}{mr} .$$

If a_G is the acceleration of G, then

$$a_G = -\frac{F}{m} .$$

If t is the time (in sec) for the wheel to stop, then

$$V_G - \frac{F}{m} t = 0 \quad \text{and}$$

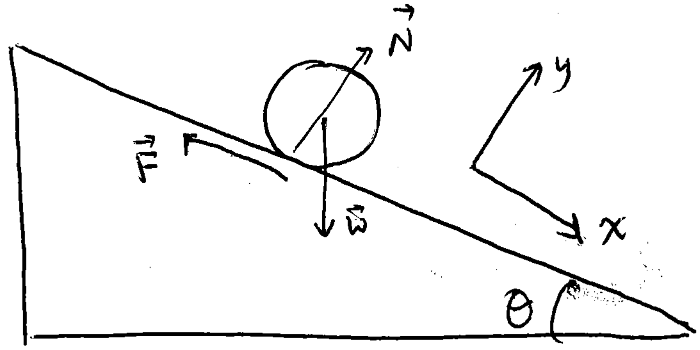
$$\omega_0 - \frac{F}{mr} t = 0 .$$

Thus,
$$t = \frac{V_G m}{F} = \frac{\omega_0 \cdot mr}{F} \quad \Rightarrow$$

$$\omega_0 = V_G / r .$$

The initial backspin is $\omega_0 = V_G/r$ rad/s.

5.



Let $\vec{F}$ be the frictional force of the ramp on the ball.

Let $\vec{N}$ " " normal force of the ramp on the ball.

Then

$$\sum F_y = m(a_G)_y \Rightarrow N = mg\cos\theta.$$

$$\sum M_G = I_G \alpha \Rightarrow Fr = \tfrac{2}{5}mr^2\alpha \Rightarrow F = \tfrac{2}{5}mr\alpha$$

$$\sum F_x = m(a_G)_x \Rightarrow mg\sin\theta - F = m(a_G)_x \Rightarrow$$

$$mg\sin\theta - \tfrac{2}{5}mr\alpha = mr\alpha \Rightarrow$$

$$g\sin\theta = \tfrac{7}{5}r\alpha, \text{ and}$$

$$F = \tfrac{2}{5}mr\alpha = \tfrac{2}{5}\left(\tfrac{5}{7}mg\sin\theta\right) = \tfrac{2}{7}mg\sin\theta.$$

When the ball is on the verge of slipping,

$$u_s = \frac{F}{N} = \frac{2}{7}\tan\theta, \text{ so}$$

$$\theta = \tan^{-1}\left(\tfrac{7}{2}u_s\right) \text{ is the maximum}$$

angle of the ramp for the ball to roll w/o slipping.