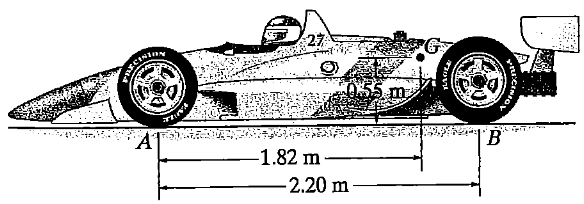
Engineering 215 Test 2 Name: ________________

Directions: Show all work and be neat and organized to receive credit. Define all variables you introduce and be sure to distinguish between scalars and vectors. You may show coordinate systems on this paper, but do all your work on separate paper.

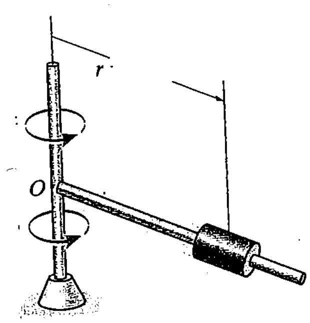
1. (20 points) The race car below has rear-wheel drive and the front tires are free to roll. The coefficients of static and kinetic friction between the wheels and the road are $\mu_s = 0.8$ and $\mu_k = 0.6$, respectively. Neglect the mass of the tires.

a. If the racer tries to accelerate too quickly and the rear wheels slip, determine the acceleration of the race car.

b. Determine the greatest possible acceleration of the race car so that its front tires do not leave the ground and the rear wheels do not slip.



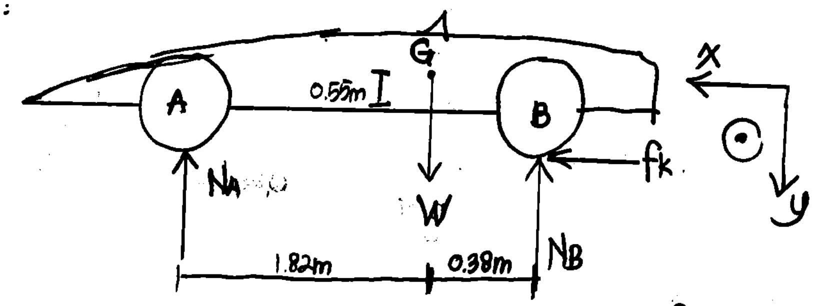
2. (20 points) A spool of mass m kilograms is initially at rest on the horizontal rod at a distance of r meters from the axis of rotation. The coefficient of static friction between the spool and the rod is μ_s. If the rod is initially at rest and then starts to rotate with a constant angular acceleration of α rad/s^2, determine the time it takes before the spool begins to slide on the rod. Neglect the size of the spool.



FRONT TIRES DO NOT LEAVE GROUND AND REAR WHEEL NOT SLIP (SO NO f_s IN B)

(b) F.B.D :

(a)



$f_k = \mu_k . N_B$

$\Sigma F_y = 0$

$N_A + N_B = W$

$\Sigma M_G = I_G . \cancel{\alpha}^{\,0}$ (NO ROTATION)

$-N_A . 1.82 + N_B . 0.38 - f_k . 0.55 = 0$

$-1.82\, N_A + 0.38\, N_B - 0.6 . 0.55\, N_B = 0$

$-1.82\, N_A + 0.05\, N_B = 0$

$$N_B = \frac{1.82}{0.05} N_A$$

$$N_B = 36.4\, N_A \quad \text{(iv)}$$

PUT (iv) INTO (iii)

$N_A + N_B = W$

$N_A + 36.4\, N_A = W$

$37.4\, N_A = W$

$$N_A = \frac{W}{37.4} = 0.0267\, W$$

$N_B = 36.4 . 0.0267\, W$

$= 0.973\, W = 0.973\, m g$

$\Sigma F_x = m . a_x$

$f_k = m . a_x$

$\mu_k . N_B = m . a_x$

$\mu_k \cdot 0.973\, \cancel{m} g = \cancel{m} a$

$a = 5.73\ \frac{m}{s^2}$

$a_{max} = \underline{5.73\ m/s^2} \longleftarrow$ m/s^2

THE GREATEST POSSIBLE ACCELERATION OF THE CAR SO THE FRONT TIRES DO NOT LEAVE GROUND AND REAR WHEELS DO NOT SLIP IS $\underline{\underline{5.73\ m/s^2}}$

~~This is part (a)~~

Need to verify

$f_s \leq \mu_s N_B$.

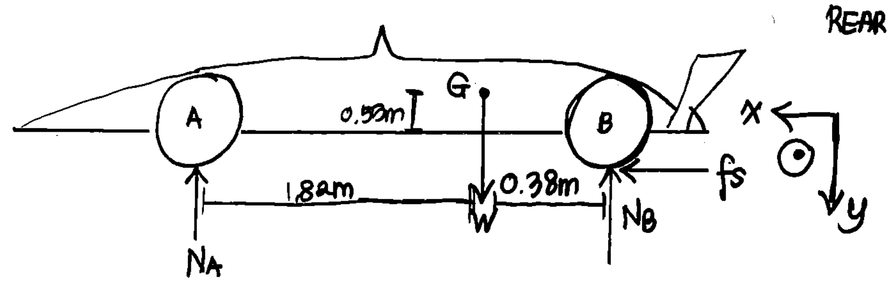
① FIND: a_G.

a_{max}

ⓐ F.B.D. REAR WHEEL DRIVE

ⓑ

REAR WHEEL SLIP



LET N_A BE NORMAL FORCE OF THE GROUND ON FRONT WHEEL A (IN NEWTON)

LET N_B BE NORMAL FORCE OF THE GROUND ON REAR WHEEL B (IN NEWTON)

LET W BE THE WEIGHT OF THE RACE CAR (IN NEWTON)

LET m BE THE MASS OF THE RACE CAR (IN kg)
LET a_x BE THE ACCELERATION IN x AXIS (in m/s^2)
LET fs BE THE STATIC FRICTION OF THE GROUND ON REAR WHEEL B (IN NEWTON)
LET fk BE THE KINETIC FRICTION ——— " ———
LET a_G BE THE ACCELERATION OF CENTER OF MASS OF RACE CAR (IN m/s^2)
LET a_{max} BE THE GREATEST POSSIBLE OF ACCELERATION OF THE RACE CAR (IN m/s^2)

$\Sigma F_y = m.\cancel{a_y}^{\,0}$

$W - N_A - N_B = 0$

$N_A + N_B = W$ ——— (i)

$\Sigma M_G = I_G.\cancel{\alpha}^{\,0}$

$-N_A . 1.82 + N_B . 0.38 - f_s . 0.55 = 0$

$-1.82\ N_A + 0.38\ N_B - \mu_s . N_B . 0.55 = 0$

$-1.82\ N_A + 0.38\ N_B - 0.8\ N_B . 0.55 = 0$

$-1.82\ N_A + 0.38\ N_B - 0.44\ N_B = 0$

$-1.82\ N_A - 0.06\ N_B = 0$

$N_A = -\frac{0.06}{1.82} N_B = -0.0330\ N_B$ ——— (ii)

PUT (ii) BACK INTO (i)

$N_A + N_B = W$

$-0.0330\, N_B + N_B = W$

$N_B = \dfrac{W}{0.967} = 1.034W$

$N_A = -0.0341W$

N_A CANNOT BE NEGATIVE

N_A HAS TO BE 0, FRONT WHEEL IS ALMOST NOT TOUCHING THE GROUND

$N_A = 0$

$\Sigma F_y = 0$

$N_B = W$

$\Sigma M_G = I_G \cancel{\alpha}^{\,0}$

$N_B \cdot 0.38 - f_s \cdot 0.55 = 0$

LET g BE THE ACCELERATION DUE TO GRAVITY (IN m/s^2)

$f_s = \dfrac{0.38\, N_B}{0.55} = 0.691\, N_B$

$= 0.691\, W$

$= 0.691\, mg$ $\leq .8 N_B$ (need to verify)

~~This is perfect~~).

$\Sigma F_x = m \cdot a_G$

$f_s = m \cdot a_G$

$0.691\, \cancel{m} g = \cancel{m} \cdot a_G$

$a_G = 0.691\, g$

$= 0.691 \cdot 9.81\ m/s^2$

$= \underline{6.78\ m/s^2}$ ← m/s^2

THE ACCELERATION OF THE RACE CAR IS $\underline{\underline{6.78\ m/s^2}}$

2) GIVEN: m kg $\quad\quad$ α constant = $\alpha \frac{rad}{s^2}$

r meters $\quad\quad$ $\dot{r}=0$

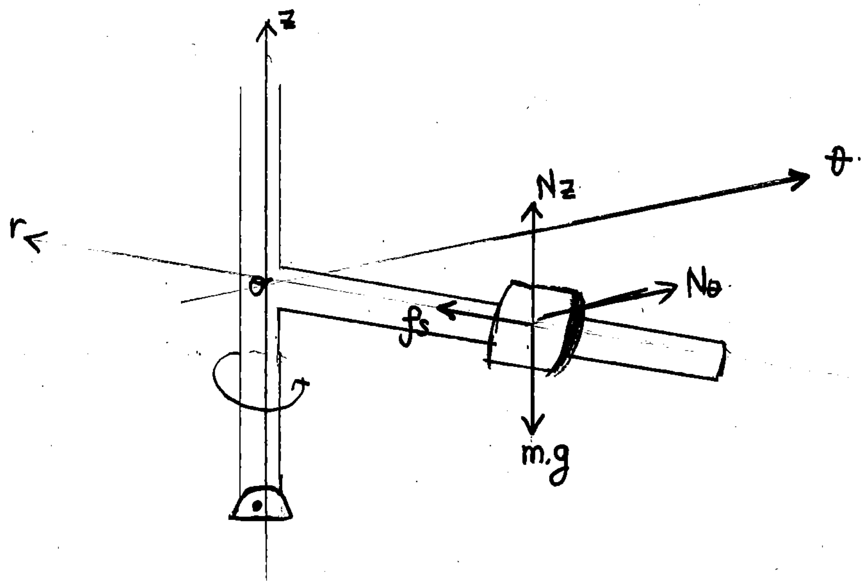
μ_s $\quad\quad$ $\ddot{r}=0$

$\omega_0=0$

FIND: TIME BEFORE SPOOL BEGINS TO SLIDE ON THE ROD

SOLUTION:

F.B.D:



WHEN THE SPOOL BEGIN TO SLIDE, THERE IS STATIC FRICTIONAL FORCE, AND r IS NOT CHANGING

LET THERE ARE 3 AXIS ; z, θ, AND r

LET N_z BE z COMPONENT OF NORMAL FORCE OF ROD ON SPOOL (IN NEWTON)

LET N_θ BE θ COMPONENT OF NORMAL FORCE OF ROD ON SPOOL (IN NEWTON)

LET N_{net} BE THE TOTAL NORMAL FORCE OF THE ROD ON THE SPOOL (IN NEWTON)

LET g BE THE ACCELERATION DUE TO GRAVITY (IN m/s^2)

LET f_s BE STATIC FRICTIONAL FORCE OF THE ROD ON THE SPOOL (IN NEWTON)

LET a_θ BE THE θ COMPONENT OF ACCELERATION (IN m/s^2)

LET a_z BE THE z COMPONENT OF ACCELERATION (IN m/s^2)

LET a_r BE THE r ——— " ———

LET t BE THE TIME IT TAKES BEFORE THE SPOOL BEGIN TO SLIDE ON ROD (IN SECOND)

LET ω_0 BE THE INITIAL ANGULAR SPEED OF SPOOL ($\frac{rad}{s}$)

LET ω_f BE ANGULAR SPEED OF SPOOL AFTER t seconds ($\frac{rad}{s}$)

LET $\dot{\theta} = \omega$ BE THE ANGULAR SPEED OF THE ROD (IN rad/s)

LET $\ddot{\theta}$ BE THE ANGULAR ACCELERATION OF THE ROD (IN rad/s²)

LET r BE THE DISTANCE OF THE SPOOL FROM THE CENTRAL AXIS (IN METERS)

LET $\dot{r}$ BE THE RATE OF CHANGE OF DISTANCE OF SPOOL FROM CENTRAL AXIS (IN METERS/s)

LET $\ddot{r}$ BE THE RATE OF CHANGE OF $\dot{r}$ (IN m/s)

$$\Sigma F_z = m.\cancel{a_z}^{\,0}$$

$$N_z - mg = 0$$

$$N_z = m.g$$

$$\Sigma F_r = m.\ a_r$$

$$f_s = m.(\cancel{\ddot{r}}^{\,0} + r(\dot{\theta})^2)$$ ✓

$$\mu_s.\ N_{net} = m.(\ r.(\dot{\theta})^2) \quad \text{—— (i)}$$

$$\Sigma F_\theta = m.\ a_\theta$$

$$N_\theta = m.(r.\ddot{\theta} + \cancel{2\dot{r}.\dot{\theta}}^{\,0})$$ ✓

$$N_\theta = m.(r\alpha + 0)$$

$$N_\theta = m.r.\alpha \quad \text{—— (ii)}$$

$$N_{net} = \sqrt{N_z^2 + N_\theta^2} = \sqrt{(m.g)^2 + (m.\alpha.r)^2}$$ ✓

$$= m\sqrt{g^2 + (\alpha r)^2}$$

SINCE IT IS A CONSTANT ANGULAR ACCELERATION,

$$\omega_f = \omega_0 + \alpha t$$ ✓

$$= 0 + \alpha.t$$

$$= \alpha.t$$

PUT N_{net} INTO (i)

$$\mu_s.\cancel{m}\sqrt{g^2 + (\alpha r)^2} = \cancel{m}.(r.\ \omega^2)$$

$$\frac{\mu_s}{r}\sqrt{g^2 + (\alpha r)^2} = \omega^2$$

$$\omega_f = \sqrt{\frac{\mu_s}{r}\sqrt{g^2 + (\alpha r)^2}}$$ → AT THIS ANGULAR SPEED, SPOOL BEGINS TO SLIDE ON THE ROD

$$\alpha.t = \sqrt{\frac{\mu_s}{r}\sqrt{g^2 + (\alpha r)^2}}$$ ✓

$$\underline{\underline{t = \frac{1}{\alpha}\sqrt{\frac{\mu_s}{r}\sqrt{g^2 + (\alpha r)^2}}\ s}}$$ ← SECONDS

THE TIME IT TAKES BEFORE THE SPOOL BEGINS TO SLIDE ON THE ROD IS $\underline{\underline{\frac{1}{\alpha}\sqrt{\frac{\mu_s}{r}\sqrt{g^2+\alpha^2 r^2}}}}$ SECONDS

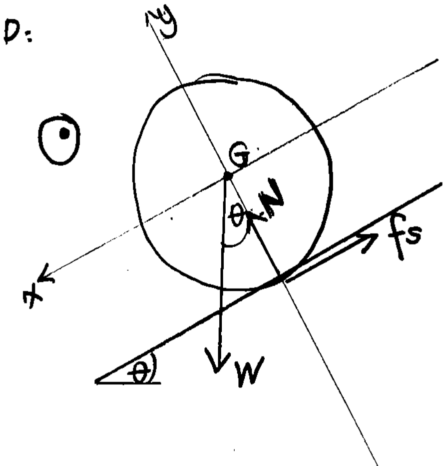
(3) GIVEN: MASS WHEEL = m kg ANGLE θ.

RADIUS: r meters COEFFICIENT STATIC μ_s

GYRATION: k meters ROLLING WITHOUT SLIPPING SO $f_s \leq \mu_s . N$

FIND: θ_{max} SO WHEEL ROLLS WITHOUT SLIPPING

SOLUTION: F.B.D:



TO MAXIMIZE THE θ MEANS WE HAVE TO MAXIMIZE THE f_s ALSO, SO $f_s = \mu_s . N$,
SO THE BALL CAN REACH THE LARGEST FRICTIONAL FORCE WITHOUT SLIPPING

LET N BE THE NORMAL FORCE OF INCLINED PLANE ON THE WHEEL (IN NEWTON)

LET W BE THE WEIGHT OF THE WHEEL (IN NEWTON)

LET f_s BE THE STATIC FRICTION OF THE PLANE ON THE WHEEL (IN NEWTON)

LET a_x BE THE X COMPONENT OF WHEEL ACCELERATION of CM (IN m/s^2)

LET a_y BE THE y COMPONENT OF WHEEL ACCELERATION of CM (IN m/s^2)

LET a_G BE ACCELERATION OF CENTER OF MASS (IN m/s^2)

LET θ_{max} BE THE MAX VALUE OF θ TO MAKE THE WHEELS ROOLS DOWN THE PLANE W/O SLIPPING (in DEGREE)

$$\Sigma F_y = m . \cancel{a_y} \to 0$$

$$N - W.\cos\theta = 0$$

$$N = W.\cos\theta$$

$$N = m.g.\cos\theta_{max}$$

$$\Sigma M_G = I_G . \alpha$$

$$f_s . r = m . k^2 . \alpha$$

f_s → LARGEST FRICTIONAL VALUE IS $\mu_s . N$

$$\mu_s . N . r = m . k^2 . \alpha$$

$$\mu_s . \cancel{m} . g . r . \cos\theta_{max} = \cancel{m} . k^2 . \alpha$$

$$\alpha = \frac{\mu_s . g . r . \cos\theta_{max}}{k^2}$$

$$a_G = \alpha . r = \mu_s . g . \cos\theta_{max} . \frac{r^2}{k^2}$$

$a_x = a_G.$

$$a_x = \mu s \cdot g \cdot \cos\theta_{max} \cdot \frac{r^2}{k^2}$$

$$\Sigma F_x = m \cdot a_x$$

$$W \cdot \sin\theta_{max} - f_s = m \cdot \mu s \cdot g \cdot \cos\theta_{max} \cdot \frac{r^2}{k^2}$$

$$m \cdot g \cdot \sin\theta_{max} - \mu s \cdot m \cdot g \cdot r \cdot \cos\theta_{max} = \mu s \cdot m g \cdot \frac{r^2}{k^2} \cos\theta_{max}$$

DIVIDE BOTH SIDE WITH $\cos\theta$.

$$m \cdot g \cdot \tan\theta_{max} - \mu s \cdot m \cdot g \cdot r = \mu s \cdot m \cdot g \cdot r \left(\frac{r}{k^2}\right)$$

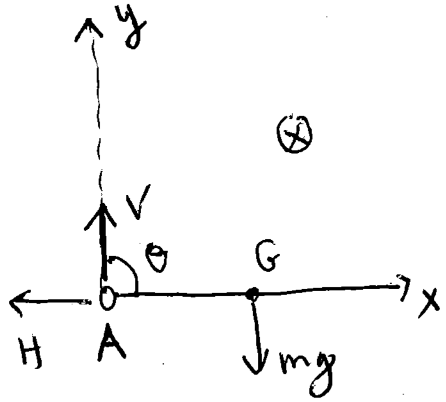
$$\cancel{m \cdot g} \cdot \tan\theta_{max} = \mu s \cdot \cancel{m g} \cdot r \left(\frac{r^2}{k^2} + 1\right)$$

$$\tan\theta_{max} = \mu s \cdot r \left(\frac{r^2}{k^2} + 1\right)$$

$$\theta_{max} = \underline{\underline{\arctan\left(\mu s\, r \left(1 + \frac{r^2}{k^2}\right)\right)^{\circ}}} \longleftarrow \text{DEGREE}$$

THE MAXIMUM VALUE OF θ THAT THE WHEEL ROLLS DOWN THE PLANE WITHOUT SLIPPING is $\underline{\underline{\left(\arctan\left(\mu s \cdot r\left(1 + \frac{r^2}{k^2}\right)\right)\right)^{\circ}}}$

4.



Let θ be the angle thru which the bar has rotated.

Let H be the horizontal component of the force of the pin on the rod when $\theta = \pi/2$. (N)

Let V be the vertical component of the force of the pin on the rod when $\theta = \pi/2$. (N)

Then $\quad \Sigma M_A = I_A \alpha \quad$ (α = angular acceleration of rod when $\theta = \frac{\pi}{2}$) (rad/s^2)

$$\Rightarrow \quad mg \cdot \frac{L}{2} = \frac{1}{3} m L^2 \alpha$$

$$\Rightarrow \quad \alpha = \frac{3}{2} \cdot \frac{g}{L}$$

If a_G is the acceleration of G when $\theta = \frac{\pi}{2}$ (in m/s^2), then

$$\Sigma F_y = m(a_G)_y \Rightarrow$$

$$-mg + V = -m\frac{L}{2}\alpha = -\frac{3}{4}mg \quad \Rightarrow \quad V = \frac{1}{4}mg.$$

At a general angle θ,

$$\Sigma M_A = I_A \alpha \Rightarrow$$

$$mg \cdot \frac{L}{2} \cdot \sin\theta = \frac{1}{3} m L^2 \alpha \quad \Rightarrow \quad \alpha = \frac{3}{2} \cdot \frac{g}{L} \cdot \sin\theta,$$

where α is now the angular acceleration (rad/s^2) of the rod at the angle θ. Thus,

$$\frac{d\omega}{dt} = \frac{d\omega}{d\theta} \cdot \omega = \frac{3}{2}\frac{g}{L}\sin\theta, \text{ and}$$

$$\int_0^{\omega_f} \omega \, d\omega = \int_0^{\pi/2} \frac{3}{2}\frac{g}{L}\sin\theta \, d\theta \Rightarrow \omega_f^2 = 3 \cdot \frac{g}{L}, \text{ where}$$

ω_f is the angular velocity of the rod (rad/s) at $\theta = \pi/2$.

Then

$$\sum F_x = m(a_G)_x \Rightarrow$$

$$-H = -m \cdot \frac{L}{2} \cdot \omega_f^2 = -m \cdot \frac{L}{2} \cdot \frac{3g}{L} = -\frac{3}{2}mg$$

So $H = \frac{3}{2}mg$

Thus, the force of the pin on the rod at $\theta = \frac{\pi}{2}$ is

$$\left\langle -\frac{3}{2}mg, \frac{1}{4}mg \right\rangle,$$

so the force of the rod on the pin at $\theta = \frac{\pi}{2}$

is

$$\left\langle \frac{3}{2}mg, -\frac{1}{4}mg \right\rangle.$$

<u>Note</u>: Instead of integrating, you could use the work - kinetic energy theorem to show

$$\omega_f^2 = \frac{3g}{L}.$$