Some Interesting Problems – Set 4

1. (Math 80) If you drive *m* miles in *h* hours and arrive half an hour early, how fast should you drive to arrive on time?

2. (Math 99) If 500 pounds of mush will feed 20 pigs for a week, how long will 200 pounds of mush feed 14 pigs?

3. (Math 99) Allan starts painting a room at 1:00 PM. At 3:00 PM Betty starts to help and they finish the job at 7:00 PM. The next day Betty starts putting on a second coat of paint at 1:00 PM. Allan joins her at 4:00 PM and they finish the job at 7:00 PM. The following day they both start putting on a third coat of paint at 1:00 PM. What time do they finish?

4. (Math 110) Let $f(x)$ denote the greatest integer less than or equal to x. Let $g(x)$ denote the least integer greater than or equal to x. Express f in terms of g.

5. (Math 110) Prove that the function $f(x) = ln\ (x + \sqrt{x^2+1})$ is odd.

6. (Math 110) Let C denote the unit circle centered at the origin and let $P(a,b)$ be a point in the exterior of C. Let A and B denote the points of tangency of the two tangents to C passing through P. Prove that an equation of the line through A and B is given by $ax+by = 1$.

7. (Math 110) Find the radius of the largest circle that can be dropped into the parabola *y=x^2* and touch the origin.

8. (Math 120) Solve the equation $2\sqrt{x+1} = 3x + 5$. Check your solutions.

9. (Math 120) Find all values of θ, $0 \leq \theta < 2\pi$, such that $\left(\frac{16}{81}\right)^{\sin^2\theta} + \left(\frac{16}{81}\right)^{\cos^2\theta} = \frac{26}{27}$.

10. (Math 124) A sprinkler is located 1 foot from a wall and rotates at the rate of 1 radian/sec. The sprinkler sends a horizontal stream of water toward the wall at the constant rate of 1 gallon/sec. Ignore the effects of gravity and assume the water travels at the constant speed of c ft/sec.

   a. Find the location of the point on the wall that gets wet first.
   b. Find the linear density of water on the wall at a point x feet from the point on the wall nearest the sprinkler. Assume that the sprinkler makes one revolution. Which part of the wall gets most wet?
   c. Use part b. to prove that $\frac{d}{dx}(tan^{-1}\,x) = 1/(1+x^2)$.

11. Math (126) Prove that the logarithmic spiral $r=e^{k\theta}$ is equiangular. That is, if P is a point on the spiral, and O is the origin, prove that the ray OP intersects the spiral in a constant angle. Express this angle in terms of k.

12. (Math 207) Prove that the only equiangular plane curves are logarithmic spirals, circles centered at the origin, and lines passing through the origin.