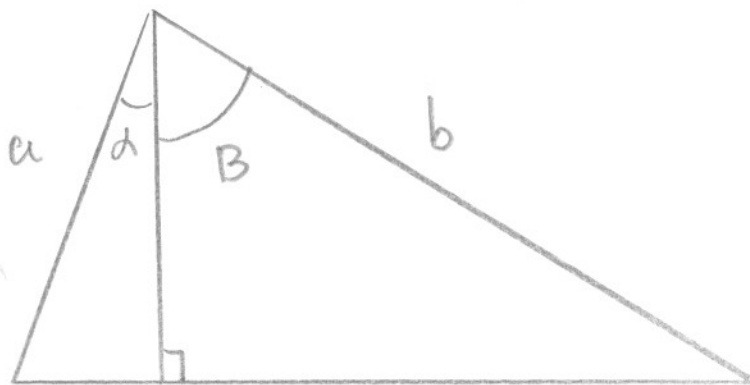


Some Interesting Problems - Set 2

- (Math 99) If the roots of  $x^2 + bx + c$  are the squares of the roots of  $x^2 + x + 1$ , find the values of  $b$  and  $c$ .
- (Math 99) Let  $f(x) = ax^7 + bx^5 + cx - 5$ . If  $f(-7) = 7$ , find  $f(7)$ .
- (Math 110) If a polynomial has a remainder of 3 when divided by  $(x - 1)$  and a remainder of 5 when divided by  $(x - 3)$ , find the remainder when the polynomial is divided by  $(x - 1)(x - 3)$ .
- (Math 110) Find all values of  $d$  such that the graph of the quadratic function  $f(x) = x(x + d) + 2d$  has its vertex on the  $x$ -axis.
- (Math 120) Compute areas using the figure below to prove that  $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$ .



- (Math 126) We are probably all familiar with the trigonometry problem where you measure the angle of elevation to the top of a building and then move a measured distance directly away from the building and measure the new angle of elevation. We are then asked to compute the height of the building. Here is a variation of that problem. You measure the angle of elevation to the top of the building to be  $\theta_1$ . You then walk  $d$  feet in a given direction (not necessarily directly away from the building) and measure the new angle of elevation to be  $\theta_2$ . You then walk an additional  $d$  feet in the same direction and measure the new angle of elevation to be  $\theta_3$ . Find the height of the building.
- (Math 211) A group of 100 passengers with boarding passes begins to board a plane. The first passenger drops his boarding pass and sits in a random seat. Every other passenger sits in her assigned seat if it is available, otherwise sits in a random seat.
  - Find the probability that the last passenger to enter the plane sits in her assigned seat.
  - Find the probability that the  $k^{\text{th}}$  passenger sits in her assigned seat.
  - What is the expected number of passengers who sit in their assigned seats?