

## Some Interesting Problems - Set 1

I thought that every couple of weeks I would try to compile some interesting problems that I have recently run across and share them with everyone. Most of these should be accessible to some of our students. Please let me know if you are not interested in receiving these and I will take you off the distribution list.

Fred

1. (Math 80) Participation in the local soccer league is up 10% from last year. If the number of girls increased 20% and the number of boys increased 5%, what fraction of the participants are currently girls?

2. (Math 99, Math 110) The graph of the function  $y = x^2 + 2ax + a$  is a parabola for each real value of  $a$ . Prove that each of these parabolas passes through a common point and determine the coordinates of that point.

3. (Math 110) Let  $f$  be a real-valued function such that

$$f(x) + 2f(2002/x) = 3x,$$

for  $x > 0$ . Find  $f(2)$ .

4. (Math 110) Let  $f(x) = kx^3 + 2k^2x^2 + k^3$ . Find the sum of all real numbers  $k$  such that  $x - 2$  is a factor of  $f(x)$ .

5. (Math 120) Simplify the function  $f(x) = \sqrt{\sin^4 x + 4 \cos^2 x} - \sqrt{\cos^4 x + 4 \sin^2 x}$  as fully as possible.

6. (Math 124) Two cars are traveling on straight roads that intersect at right angles. When Car A is at the intersection, Car B is  $d$  miles from the intersection and moving toward the intersection. If the cars are moving with constant speeds of  $v_A$  and  $v_B$  miles per hour, find the minimum distance between the cars.

7. (Math 126) Let  $C_1$  be the cone defined by the equation  $c_1 z^2 = x^2 + y^2$  and let  $C_2$  be the cone defined by  $c_2 z^2 = (x - 1)^2 + y^2$ , where  $c_1$  and  $c_2$  are non-zero constants. Let  $K$  be the intersection of  $C_1$  and  $C_2$ . Prove that if  $c_1 = c_2$ , then  $K$  is a hyperbola. If  $c_1 \neq c_2$ , prove that the orthogonal projection of  $K$  onto the  $xy$ -plane is a circle. *Note:* In the case where  $c_1 \neq c_2$ , the result implies that  $K$  lies on a cylinder whose axis of symmetry is parallel to the  $z$ -axis. It would be nice to give an explicit description of  $K$  in this case, but I have been unable to come up with a satisfactory answer. I would like to know of any results that you come up with. For example, does  $K$  lie in a plane?