

Essential Academic
Learning Requirements

Mathematics

Introduction to Mathematics

Mathematics for Today and Tomorrow

Mathematics continues to grow at a rapid rate, spreading into new fields and creating new applications in its open-ended search for patterns. Several factors—growth of technology, increased applications, impact of computers, and expansion of mathematics itself—have combined in the past century to extend greatly both the scope and the application of the mathematical sciences. The changes must be reflected in the schools if our students are to be well prepared for tomorrow’s world.

What is Mathematics?

Mathematics is a language and science of patterns. As a language of patterns, mathematics is a means for describing the world in which we live. In its symbols and vocabulary, the language of mathematics is a universal means of communication about relationships and patterns.

As a science of patterns, mathematics is a mode of inquiry that reveals fundamental understandings about order in our world. This mode of inquiry relies on logic and employs observation, simulation, and experimentation as means of challenging and extending our current understanding.

Toward a Deeper Study of Important Mathematics

More than at any other time in history, society is placing demands on citizens to interpret and use mathematics to make sense of information and complex situations. Computers and other technologies have increased our capacities for dealing with numbers for collecting, organizing, representing, and analyzing data. Tables, lists of numbers, graphs of data, and statistics summarizing information occur in every form of the media.

To be well informed as adults and to have access to desirable jobs, students today require an education in mathematics that goes far beyond what was needed by students in the past. All students must develop and sharpen their skills; deepen their understanding of mathematical concepts and processes; and hone their problem-solving, reasoning, and communication abilities while using mathematics to make sense of and to solve compelling problems. All students need a deep understanding of mathematics; for this to occur, rigorous mathematical content must be reorganized, taught, and assessed in a problem-solving environment. For students to develop this deeper level of understanding, their knowledge must be connected to a variety of ideas and skills across topic areas and grade levels in mathematics to other subjects taught in school as well as to situations outside the classroom.

Essential Academic Learning Requirements—Mathematics

1. The student understands and applies the concepts and procedures of mathematics.

To meet this standard, the student will:

- 1.1. Understand and apply concepts and procedures from number sense—number and numeration, computation, and estimation.
- 1.2. Understand and apply concepts and procedures from measurement—attributes and dimensions, approximation and precision, and systems and tools.
- 1.3. Understand and apply concepts and procedures from geometric sense—properties and relationships and locations and transformations.
- 1.4. Understand and apply concepts and procedures from probability and statistics—probability, statistics, and prediction and inference.
- 1.5. Understand and apply concepts and procedures from algebraic sense—patterns, representations, and operations.

2. The student uses mathematics to define and solve problems.

To meet this standard, the student will:

- 2.1. Investigate situations by searching for patterns and using a variety of approaches.
- 2.2. Formulate questions and define the problem.
- 2.3. Construct solutions by organizing the necessary information and using appropriate mathematical tools.

3. The student uses mathematical reasoning.

To meet this standard, the student will:

- 3.1. Analyze information from a variety of sources; use models, known facts, patterns and relationships to validate thinking.
- 3.2. Predict results and make conjectures based on analysis of problem situations.
- 3.3. Draw conclusions and verify results—support mathematical arguments, justify results, and check for reasonableness of solutions.

4. The student communicates knowledge and understanding in both everyday and mathematical language.

To meet this standard, the student will:

- 4.1. Gather information—read, listen, and observe to access and extract mathematical information.
- 4.2. Organize and interpret information.
- 4.3. Represent and share information—express and explain mathematical ideas using language and notation in ways appropriate for audience and purposes.

5. The student understands how mathematical ideas connect within mathematics, other subject areas, and real-life situations.

To meet this standard, the student will:

- 5.1. Relate concepts and procedures within mathematics—use conceptual and procedural understandings among content strands and use equivalent models and representations.
- 5.2. Relate mathematical concepts and procedures to other disciplines—identify and use mathematical patterns, thinking, and modeling in other subject areas.
- 5.3. Relate mathematical concepts and procedures to real-life situations—understand the connections between mathematics and problem-solving skills used every day at work and at home.

ESSENTIAL ACADEMIC LEARNING REQUIREMENTS—MATHEMATICS

1. The student understands and applies the concepts and procedures of mathematics.

To meet this standard, the student will:

BENCHMARK 1—GRADE 4	BENCHMARK 2— GRADE 7	BENCHMARK 3—GRADE 10
1.1. Understand and apply concepts and procedures from number sense.		
Number and Numeration		
Demonstrate understanding of whole and fractional numbers and place value in whole numbers using objects, pictures, or symbols.	Demonstrate understanding of integers, fractions, decimals, percents, place value of decimals, and properties of the rational number system using pictures and symbols.	Understand and use properties and symbolic representations of rational numbers, powers, and roots.
Identify, compare, and order whole numbers and simple fractions.	Compare and order integers, fractions, and decimals.	Compare and order rational numbers, powers, and roots.
Demonstrate an understanding of the properties of whole numbers.	Understand the concepts of prime and composite numbers, factors and multiples, and divisibility rules.	Understand concepts of and use processes involving prime and composite numbers, factors and multiples, and divisibility.
	Understand and apply the concepts of ratio and direct proportion.	Understand and apply the concepts of ratio and both direct and inverse proportion.
Computation		
Show understanding of whole number operations (+, -, ×, ÷) using blocks, sticks, beans, pictures, symbols, etc.	Understand operations on nonnegative rational numbers.	Understand operations on rational numbers, powers, and roots.
Add, subtract, multiply, and divide whole numbers.	Add, subtract, multiply, and divide nonnegative fractions and decimals using rules for order of operation.	Compute with rational numbers, powers, and roots.
Use mental arithmetic, pencil and paper, or calculator as appropriate to the task involving whole numbers.	Use mental arithmetic, pencil and paper, calculator, or computer as appropriate to the task involving nonnegative rational numbers.	Use mental arithmetic, pencil and paper, calculator, or computer as appropriate to the task involving real numbers.
Estimation		
Identify situations involving whole numbers in which estimation is useful.	Identify situations involving nonnegative rational numbers in which estimation is sufficient and computation is not required.	Identify situations involving rational numbers, powers, and roots in which estimation is sufficient and computation is not required.
Use estimation to predict computation results and to determine the reasonableness of answers, <i>for example, estimating a grocery bill.</i>	Use estimation to predict computation results and to determine the reasonableness of answers involving nonnegative rational numbers, <i>for example, estimating a tip.</i>	Use estimation to predict computation results and to determine the reasonableness of answers involving real numbers, <i>for example, estimating.</i>

ESSENTIAL ACADEMIC LEARNING REQUIREMENTS—MATHEMATICS

1. The student understands and applies the concepts and procedures of mathematics (continued).

To meet this standard, the student will:

BENCHMARK 1—GRADE 4	BENCHMARK 2—GRADE 7	BENCHMARK 3—GRADE 10
1.2. Understand and apply concepts and procedures from measurement.		
Attributes and Dimensions		
Understand concepts of perimeter, area, and volume.	Understand the concepts of and the relationships among perimeter, area, and volume and how changes in one dimension affect perimeter, area, and/or volume.	Understand how changes in dimension affect perimeter, area, and volume.
Use directly measurable attributes <i>such as length, perimeter, area, volume/capacity, angle, weight/mass, time, money, and temperature to describe and compare objects.</i>	Measure objects and events directly or using indirect methods <i>such as calculating and applying procedures for determining perimeter, area, and volume.</i>	Measure objects and events directly or use indirect methods such as finding the volume of a cone given its height and diameter.
	Understand the concept of rate and how to calculate rates and determine units.	Calculate rate and other derived and indirect measurements.
Approximation and Precision		
Understand that measurement is approximate.	Understand that precision is related to the unit of measurement used and the calibration of the measurement tool.	Understand precision and accuracy of measurement are affected by measurement tools and calculating procedures.
Know when to estimate and use estimation to determine when measurements are reasonable or to obtain approximations, for example, estimating the length of the playground by pacing it off.	Know when to estimate and use estimation to obtain reasonable approximations, <i>for example, estimating the length and width of the playground to approximate its area.</i>	Know when to estimate and use estimation to obtain reasonable approximations, <i>for example, estimating how much paint is needed to paint the walls of a classroom.</i>
Systems and Tools		
Understand the benefits of using standard units of measurement for measuring length, area, and volume.	Understand the appropriate uses of standard units of measurement for both direct and indirect measurement.	Understand the benefits of standard units of measurement and the advantages of the metric system.
Understand appropriate units of measure for time, money, length, area, volume/capacity, weight/mass, and temperature.	Understand the relationship among units within both the U.S. and metric systems.	Compare, contrast, and use both the U.S. system and metric system.
Select and use appropriate tools for measuring time, money, length, area, volume, mass, and temperature.	Select and use tools that will provide an appropriate degree of precision, <i>for example, using meters vs. kilometer.</i>	Select and use tools that will provide an appropriate degree of precision and accuracy for the situation, <i>for example, using kilometers vs. light years.</i>

ESSENTIAL ACADEMIC LEARNING REQUIREMENTS—MATHEMATICS

1. The student understands and applies the concepts and procedures of mathematics (continued).

To meet this standard, the student will:

BENCHMARK 1—GRADE 4	BENCHMARK 2—GRADE 7	BENCHMARK 3—GRADE 10
1.3. Understand and apply concepts and procedures from geometric sense.		
Properties and Relationships.		
Use attributes of geometric shapes and properties of parallel and perpendicular to identify, name, compare, and sort geometric shapes and figures.	Use the properties and relationships of plane geometry to describe shapes and figures, <i>including angles, degrees in a circle, triangles, isosceles, equilateral, or quadrilateral.</i>	Use geometric properties and relationships to compare, contrast, describe, and classify 2- and 3-dimensional geometric figures.
Recognize geometric shapes in the surrounding environment, <i>for example, identify rectangles within windows.</i>	Identify, describe, or draw objects in the surrounding environment in geometric terms, <i>for example, producing a simple scale drawing of a classroom.</i>	Construct geometric models and scale drawings using tools as appropriate, <i>for example, building a model of a bridge.</i>
Understand concepts of symmetry, congruence, and similarity.	Understand symmetry, congruence, and similarity.	Understand and use properties of symmetry, congruence, and similarity.
Draw and build simple shapes and figures using appropriate tools, <i>such as a straightedge, ruler, protractor, or nets.</i>	Perform geometric constructions using a variety of tools and technologies <i>such as paper folding, computer software, straightedge, compass.</i>	Perform complex geometric constructions using a variety of tools and technologies <i>such as paper folding, computer software, straightedge, compass.</i>
Locations and transformations.		
Locate and describe the location of objects on a number line, map, or a coordinate grid in the first quadrant.	Identify and describe location of objects on coordinate grids in any of the four quadrants.	Understand and use coordinate grids.
Understand and draw simple geometric transformations using translations (slides), reflections (flips), or rotations (turns).	Understand and apply simple geometric transformations using combinations of translations (slides), or reflections (flips), or rotations (turns).	Understand and apply multiple geometric transformations using combinations of translations, reflections, and/or rotations.

ESSENTIAL ACADEMIC LEARNING REQUIREMENTS—MATHEMATICS

1. The student understands and applies the concepts and procedures of mathematics (continued).

To meet this standard, the student will:

BENCHMARK 1—GRADE 4	BENCHMARK 2—GRADE 7	BENCHMARK 3—GRADE 10
1.4. Understand and apply concepts and procedures from probability and statistics.		
Probability		
Understand the difference between a certain and uncertain event.	Know how to calculate numerical measures of chance for simple events.	Understand the properties of dependent and independent events.
Know how to list all possible outcomes of simple experiments.	Understand procedures for counting outcomes to determine probabilities.	Understand and use appropriate counting procedures to determine probabilities.
Understand and use experiments to investigate the probabilities of uncertain events.	Know how to conduct experiments and simulations and to compare results with mathematical expectations.	Use both experimental and theoretical methods to determine probabilities.
Statistics		
Collect data in an organized way.	Collect a random sample of data that represents a described population.	Collect data using appropriate methods and technology.
Organize and display data in numerical and graphical forms <i>such as tables, charts, pictographs, and bar graphs.</i>	Organize and display data in appropriate forms <i>such as frequency tables, circle graphs, and stem-and-leaf plots.</i>	Organize and display data in appropriate forms <i>such as tables, graphs, scatter plots, and box and whisker plots.</i>
Understand measures of central tendency, <i>such as mean, median, and mode in describing data.</i>	Calculate and appropriately use range and measures of central tendency to describe data.	Calculate and use the different measures of central tendency, variability, and range as appropriate to describe data.
Identify how data can be used to support a point of view.	Identify how statistics can be used to support different points of view.	Use statistics to support different points of view, <i>for example, in a debate or a position paper.</i>
Prediction and Inference		
Predict outcomes of simple activities and compare predictions to experimental results.	Predict outcomes of experiments and simulations and compare the predictions to experimental results.	Predict outcomes and design and conduct experiments to verify or disprove predictions.
Understand and make inferences based on experimental results using coins, number cubes, spinners, etc.	Understand and make inferences based on analysis of experimental results, statistical data, and simple graphical representations.	Understand and make inferences based on the analysis of experimental results, statistical data, and graphical representations.

ESSENTIAL ACADEMIC LEARNING REQUIREMENTS—MATHEMATICS

1. The student understands and applies the concepts and procedures of mathematics (continued).

To meet this standard, the student will:

BENCHMARK 1—GRADE 4	BENCHMARK 2—GRADE 7	BENCHMARK 3—GRADE 10
1.5. Understand and apply concepts and procedures from algebraic sense.		
Patterns		
Recognize, extend, and create patterns of numbers, shapes, or objects <i>such as beans, toothpicks, pattern blocks, cubes, and colored tiles.</i>	Recognize, extend, and create patterns and sequences.	Recognize, extend, and create complex patterns and sequences.
Write a rule for a pattern based on a single arithmetic operation between terms such as a function machine.	Represent and describe patterns with tables, graphs, and rule.	Generalize and express rules describing patterns and sequences.
Representations		
Understand equality and inequality and use =, >, and < in number sentences.	Represent equalities and inequalities symbolically using =, >, <, ≤, ≥.	Translate among tabular, symbolic, and graphical representations of relations using =, ≠, >, <, ≥, ≤.
Identify and use appropriate symbols and notations in reading and writing open sentences; for example, $3 \times \square = 18$.	Use variables to write simple expressions, equations, and inequalities, for example, $3x > 18$.	Use variables to write expressions, equations, and inequalities.
Operations		
Evaluate simple expressions using blocks, sticks, beans, pictures, etc.	Evaluate expressions and formulas.	Simplify and evaluate expressions and formulas.
Solve simple equations using blocks, sticks, beans, pictures, etc.	Solve single-variable equations.	Solve equations and inequalities.

ESSENTIAL ACADEMIC LEARNING REQUIREMENTS—MATHEMATICS

2. The student uses mathematics to define and solve problems.

To meet this standard, the student will:

BENCHMARK 1—GRADE 4	BENCHMARK 2—GRADE 7	BENCHMARK 3—GRADE 10
2.1. Investigate situations		
Search for patterns in simple situations.	Search systematically for patterns in simple situations.	Search systematically for patterns in complex situations.
Use a variety of strategies and approaches.	Develop and use a variety of strategies and approaches.	Use multiple strategies.
Recognize when information is missing or extraneous.	Identify missing or extraneous information.	Identify what information is missing or extraneous and compensate for it.
Recognize when an approach is unproductive and try a new approach.	Recognize the need to modify or abandon an unproductive approach.	Analyze an unproductive approach and attempt to modify it or try a new approach.
2.2. Formulate questions and define the problem		
Identify questions to be answered in familiar situations.	Identify questions to be answered in new situations.	Identify questions to be answered in complex situations.
Define problems in familiar situations.	Define problems in new situations.	Define problems in complex situations.
Identify what is known and unknown in familiar situations.	Identify the known and unknown in new situations.	Identify the information that is known and unknown in complex situations.
2.3. Construct solutions		
Organize relevant information.	Organize relevant information from multiple sources.	Organize and synthesize information from multiple sources.
Select and use appropriate mathematical tools.	Select and use appropriate mathematical tools.	Select and use appropriate mathematical tools.
Apply viable strategies and appropriate concepts and procedures to construct a solution.	Apply viable strategies and appropriate concepts and procedures to construct a solution.	Apply viable strategies and appropriate concepts and procedures to construct a solution.

ESSENTIAL ACADEMIC LEARNING REQUIREMENTS—MATHEMATICS

3. The student uses mathematical reasoning.

To meet this standard, the student will:

BENCHMARK 1—GRADE 4	BENCHMARK 2—GRADE 7	BENCHMARK 3—GRADE 10
3.1. Analyze information.		
Compare and interpret information in familiar situations.	Compare, contrast, and interpret information from a variety of sources.	Compare, contrast, interpret and integrate information from multiple sources.
Validate thinking using models, known facts, patterns, and relationships.	Validate thinking and mathematical ideas using models, known facts, patterns, relationships, and counter-examples.	Validate thinking and mathematical ideas using models, known facts, patterns, relationships, counter-examples, and proportional reasoning.
3.2 Predict results.		
Make conjectures based on analysis of familiar problem situations.	Make conjectures based on analysis of new problem situations.	Make and explain conjectures based on analysis of problem situations.
3.3. Draw conclusions and verify results.		
Test conjectures by finding examples to support or contradict them.	Test conjectures and explain why they are true or false.	Test conjectures by formulating a proof or by constructing a counterexample.
Support arguments and justify results.	Support arguments and justify results using evidence.	Support arguments and justify results using inductive and deductive reasoning.
Check for reasonableness of results.	Check for reasonableness of results.	Check for reasonableness of results.
Reflect on and evaluate procedures and results in familiar situations.	Reflect on and evaluate procedures and results in new problem situations.	Reflect on and evaluate procedures and results and make necessary revisions.

ESSENTIAL ACADEMIC LEARNING REQUIREMENTS—MATHEMATICS

4. The student communicates knowledge and understanding in both everyday and mathematical language.

To meet this standard, the student will:

BENCHMARK 1—GRADE 4	BENCHMARK 2—GRADE 7	BENCHMARK 3—GRADE 10
4.1. Gather information		
Develop and follow a simple plan for collecting information.	Develop and follow a plan for collecting information.	Develop or select and follow an efficient system for collecting information.
Use reading, listening, and observation to access and extract mathematical information from a variety of sources <i>such as pictures, diagrams, physical models, classmates, oral narratives, and symbolic representations.</i>	Use reading, listening, and observation to access and extract mathematical information from multiple sources <i>such as pictures, diagrams, physical models, oral narratives, and symbolic representations.</i>	Use reading, listening, and observation to access and extract mathematical information from multiple, self-selected sources <i>such as pictures, diagrams, physical models, oral narratives, and symbolic representations.</i>
Use available technology to browse and retrieve mathematical information from a variety of sources.	Choose appropriate available technology to browse, select, and retrieve relevant mathematical information from a variety of sources.	Integrate the use of a variety of available technologies to browse, select, and retrieve mathematical information from multiple sources.
4.2. Organize and interpret information		
Organize and clarify mathematical information in at least one way—reflecting, verbalizing, discussing, or writing.	Organize and clarify mathematical information by reflecting, verbalizing, discussing, or writing.	Organize, clarify, and refine mathematical information in multiple ways—reflecting, verbalizing, discussing, or writing.
4.3. Represent and share information		
Express ideas using mathematical language and notation <i>such as physical or pictorial models, tables, charts, graphs, or symbols.</i>	Clearly and effectively express or present ideas and situations using both everyday and mathematical language <i>such as models, tables, charts, graphs, written reflection, or algebraic notation.</i>	Express complex ideas and situations using mathematical language and notation in appropriate and efficient forms.
Explain or represent mathematical ideas and information to familiar people for a given purpose.	Explain or represent mathematical ideas and information in ways appropriate for audience and purpose.	Explain or represent complex mathematical ideas and information in ways appropriate for audience and purpose.

ESSENTIAL ACADEMIC LEARNING REQUIREMENTS—MATHEMATICS

5. The student understands how mathematical ideas connect within mathematics, other subject areas, and real-life situations.

To meet this standard, the student will:

BENCHMARK 1—GRADE 4	BENCHMARK 2—GRADE 7	BENCHMARK 3—GRADE 10
5.1. Relate concepts and procedures within mathematics.		
Relate conceptual and procedural understandings among familiar mathematical content strands.	Relate and use conceptual and procedural understandings among a variety of mathematical content areas.	Relate and use conceptual and procedural understandings among multiple mathematical content strands.
Recognize equivalent mathematical models and representations in familiar situations.	Relate and use different mathematical models and representations of the same situation.	Relate and use multiple equivalent mathematical models and representations.
5.2. Relate mathematical concepts and procedures to other disciplines.		
Recognize mathematical patterns and ideas in familiar situations in other disciplines.	Identify mathematical patterns and ideas in other disciplines.	Extend mathematical patterns and ideas to other disciplines.
Use mathematical thinking and modeling in familiar situations in other disciplines.	Use mathematical thinking and modeling in other disciplines.	Apply mathematical thinking and modeling in other disciplines.
Describe examples of contributions to the development of mathematics <i>such as the contributions of women, men, and different culture.</i>	Describe examples of contributions to the development of mathematics <i>such as the contributions of women, men, and different cultures.</i>	Describe examples of contributions to the development of mathematics <i>such as the contributions of women, men, and different cultures.</i>
Relate mathematical concepts and procedures to real-life situations.		
Give examples of how mathematics is used in everyday life.	Recognize the widespread use of mathematics in daily life and the extensive use of mathematics outside the classroom, <i>for example, in banking or sports statistic.</i>	Identify situations in which mathematics can be used to solve problems with local, national, or international implications <i>such as calculating resources necessary for interstate highway maintenance.</i>
Identify how mathematics is used in career settings.	Investigate the use of mathematics within several occupations/careers of interest.	Investigate the mathematical knowledge and training requirements for occupational/career areas of interest.