

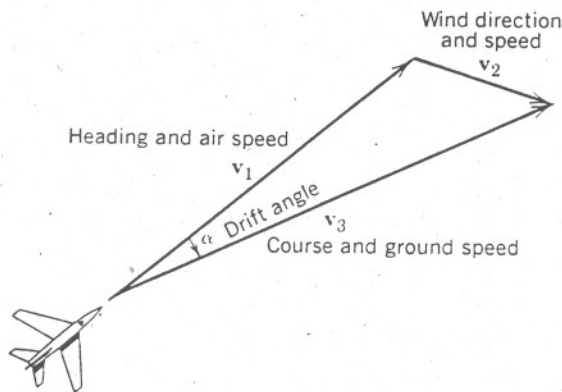
PROBLEMS CONCERNING MOTION OF A PLANE OR BOAT

USING VECTORS TO DESCRIBE MOTION

When a plane flies, its path and speed are influenced by two things -- the plane's engine and the movement of the air. The moving air can make the plane go faster, slow it down, or push the plane to the left or to the right.

The situation is similar for a motor boat or swimmer. The movement of the water will affect the speed and the direction.

This relationship is shown in the following diagram. Vector v_1 represents the effect of the plane's engine and vector v_2 represents the effect of the wind. Vector v_3 shows the actual path of the plane relative to the ground.



The length of vector v_1 represents the plane's air speed and the direction of vector v_1 shows the heading. The length of vector v_2 shows the speed of the wind and the direction of vector v_2 shows the direction of the wind. The length of vector v_3 shows the ground speed and the direction of vector v_3 shows the course.

The following terminology is often used.

Air speed Plane's speed relative to the air

Course Direction the plane or boat is moving relative to the ground.

Cross wind Wind which is neither a head wind nor a tail wind.

Drift angle Angle between the heading and the course.

Ground speed Plane's speed relative to the ground.

Heading Direction the plane or boat is pointed.

Head wind Wind blowing in direct opposition to the motion of the plane.

Northward Going toward the North. (Similar definitions for other directions.)

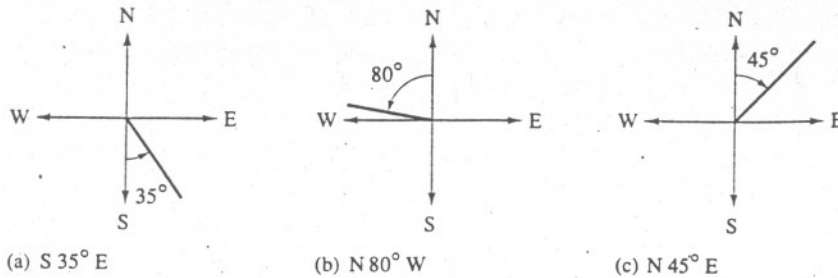
North wind Wind blowing from the North. (Similar definitions for other directions.)

Tail wind Wind blowing directly from the rear of the plane in the direction of its motion.



THE BEARING OF A LINE

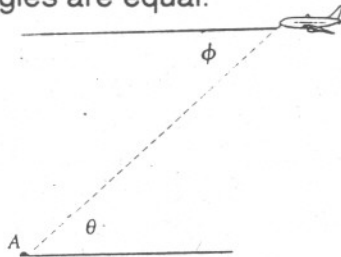
The **bearing of a line** is the acute angle formed by the north-south axis and the line. The notation used to designate the bearing of a line begins with N or S, followed by a number of degrees, and ends with E or W. Three examples are shown below.



ANGLES OF ELEVATION AND DEPRESSION

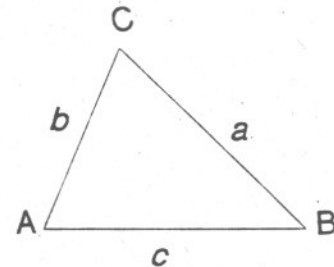
Angles are often measured with respect to a horizontal line. The **angle of elevation** is the angle from the horizontal upward to an object. The **angle of depression** is the angle from the horizontal downward to an object.

Suppose a person is standing at point A, as shown in the sketch below. If the person looks up at the plane, then angle θ is the angle of elevation. On the other hand, if the pilot looks down at the person on the ground, then angle ϕ is the angle of depression. Note that, in this case, the two angles are equal.



SOLVING OBLIQUE TRIANGLES

When working with oblique (non-right) triangles, it is generally useful to use the law of sines or cosines. Triangle ABC is oblique. The angles are named A, B, and C (upper case letters) and the sides are named a , b , and c (lower case letters). The letters are selected so that side a is opposite angle A, side b is opposite angle B, and side c is opposite angle C.



To find all the sides and angles of an oblique triangle, you need to know the measure of at least one side and two additional parts -- either two sides, two angles, or one angle and one side.

The **Law of Sines** is useful when you know two angles and any side or two sides and an angle opposite one of them.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The **Law of Cosines** is useful when you know three sides or two sides and their included angle. (Note that a similar formula could be given for b^2 or c^2 .)

$$a^2 = b^2 + c^2 - 2bc \cos A$$