RECOGNIZING AND PROVING TRIG IDENTITIES

A ***conditional equation*** is an equation which is only true for particular values of the variable. In other words, it is only true under certain conditions.

Examples: $x^2 = 9$ (Only true when $x = 3$ or $x = -3$)

$\cos \Theta = 1$ (Only true when Θ has the form $k(2\pi)$, where k is any integer, and assuming Θ is expressed in radian measure.)

An ***identity equation*** is an equation which is true for <u>all</u> defined values of the variable.

Examples: $2 + x = x + 2$

$\sin^2 \Theta + \cos^2 \Theta = 1$

Determining whether an Equation is a Conditional or an Identity Equation

Substitute any value in for the variable. If the resulting statement is *not* true, then the equation is a conditional equation. If the resulting statement is true, then likely the equation is an identity. To be more sure, substitute in another value. If that statement is also true, then it becomes even more likely that the equation is an identity. To prove that the equation is an identity, however, you must prove that it is a true statement for <u>all</u> values of the variable.

Proving that an Equation is an Identity

To prove that an equation is an identity, you must be able to form a chain of equalities from one side to the other. It is possible to work with either or both sides, but you must be sure to work with them separately. (If you worked with them together, then you would be assuming that the two sides are always equal, but that is what you want to prove.)

Form of Proof that an Equation is an Identity

There are several ways to present your work, but the following is one of the best. Suppose you were asked to prove that A = B, where A and B are different-appearing expressions. You discover you can rewrite A as C and that you can rewrite C as D. You also discover that you can rewrite B as E and that you can rewrite E as F and that you can rewrite F as D. In abbreviated form you have the following, where all letters represent some expression which is equal to the expression above it.

A	B
C	E
D	F
	D

You can summarize your work by writing the following chain of equalities:

A = C = D = F = E = B It therefore follows that A = B.

Proofs that an Equation is an Identity

Example 1 **Prove that $\cos\Theta \cot\Theta + \sin\Theta = \csc\Theta$**

$\cos\Theta \cot\Theta + \sin\Theta \csc\Theta$	$\csc\Theta$
$\cos\Theta \dfrac{\cos\Theta}{\sin\Theta} + \sin\Theta$	
$\dfrac{\cos^2\Theta}{\sin\Theta} + \dfrac{\sin^2\Theta}{\sin\Theta}$	$\dfrac{1}{\sin\Theta}$
$\dfrac{\cos^2\Theta + \sin^2\Theta}{\sin\Theta}$	
$\dfrac{1}{\sin\Theta}$	

Therefore,

$$\cos\Theta \cot\Theta + \sin\Theta \csc\Theta = \cos\Theta \frac{\cos\Theta}{\sin\Theta} + \sin\Theta = \frac{\cos^2\Theta + \sin^2\Theta}{\sin\Theta} = \frac{1}{\sin\Theta} = \csc\Theta$$

and so $\cos\Theta \cot\Theta + \sin\Theta = \csc\Theta$ for every defined value of Θ.

Example 2 **Prove that $\cos 3\Theta = 4\cos^3\Theta - 3\cos\Theta$**

$\cos 3\Theta$	$4\cos^3\Theta - 3\cos\Theta$
$\cos(2\Theta + \Theta)$	
$\cos 2\Theta \cos\Theta - \sin 2\Theta \sin\Theta$	
$(\cos^2\Theta - \sin^2\Theta)\cos\Theta - (2\sin\Theta\cos\Theta)\sin\Theta$	
$\cos^3\Theta - \sin^2\Theta\cos\Theta - 2\sin^2\Theta\cos\Theta$	
$\cos^3\Theta - (1 - \cos^2\Theta)\cos\Theta - 2(1 - \cos^2\Theta)\cos\Theta$	
$\cos^3\Theta - \cos\Theta + \cos^3\Theta - 2\cos\Theta + 2\cos^3\Theta$	
$4\cos^3\Theta - 3\cos\Theta$	

Therefore, $\cos 3\Theta = 4\cos^3\Theta - 3\cos\Theta$ for all values of Θ.

Suggestions

Begin with the more complicated expression and try to simplify it. (See Example 1)

Convert all functions to sines and cosines. (See Example 1)