Significant figures – sidebar

Why do we need to know this?

Some numbers are exactly correct. When I count my children, I know the answer is actually exactly three, not 2 or 3.5 or π. Exactly correct numbers like this, though, are the exception. Most numbers have been rounded because they are the results of measurements.

Calculators and computers can compute numbers to hundreds of decimal places. But those highly detailed numbers don't necessarily reflect reality. Every calculation that depends on a rounded number will be at least a little bit wrong. Since most of the numbers we use are rounded, we need to know just how much of that precise calculation we can trust.

What should you know?

Measurements are approximations. We can only measure according to the units that are marked on our measuring device. My bathroom scale gives me my weight in a whole number of pounds. It can't tell me if I actually weigh a few ounces more or less than the number it tells me – it just reports the whole number of pounds closest to my actual weight. Similarly, when I report my height, I should say "I'm 5 feet one inch tall *to the nearest half-inch.*" Every measurement is an approximation – in fact, every measurement is at least a little bit … wrong.

Accuracy and Precision. There are two different notions of how correct a number is. Accuracy describes how close our number is to the truth, and precision describes the amount of detail we have in our measurement. In most cases, accuracy is more important than precision. In particular, a wrong answer is just as wrong if it shows five places after the decimal.

Significant Digits. Calculators and computers can compute numbers to hundreds of decimal places. But those highly detailed numbers don't necessarily reflect reality.

Example: Every Winter break my family and I drive down to California. The trip is about 800 miles and it takes us about 18 hours of driving (including rest stops). Our average speed, then, is about 800/18 miles per hour. My calculator tells me that 800/18 is 44.444444… miles per hour. But remember that I was only approximating the number of miles and the number of hours in the first place. Do I really know my average speed out to the millionth of a mile per hour? No. With the precision of the information I started with, I can really only be sure that my average speed is about 40 miles per hour.

The significant digits of a number are the ones you know for sure (as opposed to the ones you rounded away). The number of significant digits in a number is one way to measure the precision of the measurement.

The rules about significant digits.
- Calculations never give you more significant digits than you started with. You can't magically produce precise answers from imprecise ones. So the result of a calculation has AT MOST the number of significant digits of your least precise original number. If you add $179.99 to the federal budget, the number you quote (to the nearest billion) doesn't change.
- Calculations can make your precision worse; you can lose significant digits. This is especially true when you're dividing by a rounded number or raising rounded numbers to powers.
- To keep as many significant digits, as much precision, as possible – always wait until the end of the problem to do any rounding.

Significant digits in a math class. The discussion here and in your science classes is the truth about significant digits. But significant digits are usually ignored in math classes. There is one big reason math teachers ignore significant digits – we need to see more detail in your answer so that we can tell how you got it.

In most math classes:
- Use all the digits your calculator provides and always wait until the very end of all your calculations to do any rounding.
- When you present your final answer, round it to something that makes sense. If you've found an amount of money, round it to the nearest cent. If you've computed the number of people, round it to the nearest person.
- If there's no obvious context, show at least two digits after the decimal place.
- Show your work, so that your teacher can more easily trace how you got your number.