## REFLECTIONS, SCALE CHANGES, AND TRANSLATIONS

Procedure: Begin with a function whose graph you know. Work one step at a time, replacing either x or y as described below. Start with the equation of your basic graph and rewrite it, one step at a time, until you have reached the desired equation. It is usually best to do translations last. Delay your graphing until you have created your list of equations.

| REFLECTIONS | Replacement | Effect on Graph |
| :---: | :---: | :---: |
|  | Interchange $x$ and $y$. | Reflection about the line $y=x$. |
|  | Replace x by -x. | Reflection about the Y-axis. |
|  | Replace y by -y. | Reflection about the X-axis. |
| SCALE CHANGES | Replace x by Ax where $\mathrm{A}>0$. | Horizontal scale change If $A>1$, graph shrinks toward $Y$-axis. If $A<1$, graph stretches away from $Y$-axis. |
|  | Replace y by By where $\mathrm{B}>0$. | Vertical scale change <br> If $B>1$, graph shrinks toward $X$-axis. <br> If $B<1$, graph stretches away from $X$-axis. |
| TRANSLATIONS | Replace x by $\mathrm{x}-\mathrm{C}$. | Horizontal translation <br> If $C>0$, graph goes right $C$ units. <br> If $C<0$, graph goes left $\|C\|$ units. |
|  | Replace y by y-D. | Vertical translation <br> If $D>0$, graph goes up $D$ units. <br> If $\mathrm{D}<0$, graph goes down $\|\mathrm{D}\|$ units. |

## Example 1 Graph $y=-3 x$

Basic equation: $y=3 x$

$-y=3 x$
(Same as $y=-3^{x}$ )

Reflect about the X -axis


## Example 2 Graph $x=|2 y+6|$

Basic equation: $y=|x|$
$x=|y|$
Reflect about line $y=x$

Shrink vertically (toward X-axis)
$x=|2(y+3)|$
(Same as $\mathrm{x}=|2 \mathrm{y}+6|$ )





## Example 3 Graph $y=\sin (2 x+\pi / 2)$

Basic equation: $y=\sin x$

$$
y=\sin 2 x
$$

Shrink horizontally (period changes from $2 \pi$ to $\pi$ )

$$
y=\sin (2(x+\pi / 4)
$$

Translate horizontally (left $\pi / 4$ )
(Same as $y=\sin (2 x+\pi / 2)$ )




