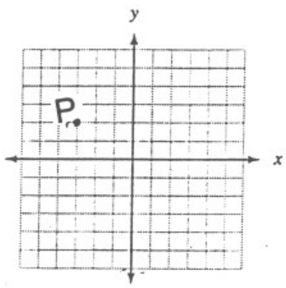
THE POLAR COORDINATE SYSTEM

Review of the Rectangular (Cartesian) Coordinate System

A rectangular coordinate system usually consists of two perpendicular lines, one horizontal and one vertical. For this discussion, the horizontal line will be called the ***X-axis*** and the vertical line will be called the ***Y-axis***. The point of intersection of these two lines is called the ***origin***. The points of the X-axis to the right of the origin represent positive numbers and the points of the X-axis to the left of the origin represent negative numbers. Similarly, the points of the Y-axis above the origin represent positive numbers and the points of the Y-axis below the origin represent negative numbers.

Every point in the rectangular plane is described by an ordered pair of real numbers, (x,y). The x and y are called the coordinates of the point. The first coordinate describes the number of units the point is to the right or left of the Y-axis and the second coordinate describes the number of units the point is above or below the X-axis. To illustrate, suppose the coordinates of point P are (-3, 2). The graph of point P is shown below.



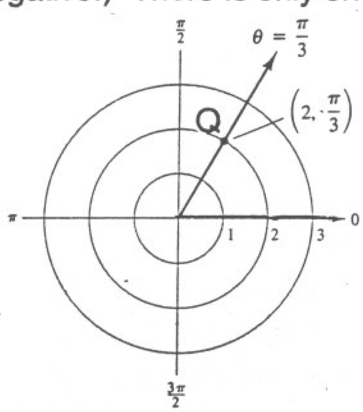
Note that every point can be described by only one ordered pair of real numbers. For example, the first coordinate of P must be -3 (although it might be written in another form, such as -6/2) and the second coordinate must be 2.

Introduction to the Polar Coordinate System

A polar coordinate system consists of a fixed point (called the ***pole*** or origin) and a ray from the origin (called the ***polar axis***). The polar axis is usually horizontal and directed toward the right.

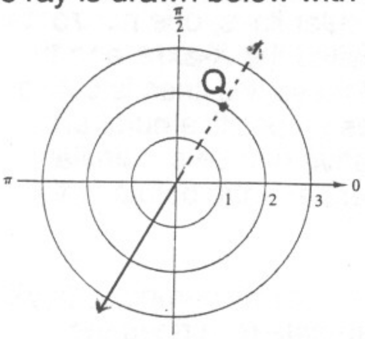
Every point in the polar coordinate system is described by an ordered pair of real numbers, (r, θ). The first coordinate describes the point's distance from the pole and the second coordinate describes the angle formed with the polar axis.

To illustrate, suppose Q has coordinates $(2, \pi/3)$. Point Q must be two units from the origin and Q must be on the ray which forms an angle of $\pi/3$ with the polar axis. (Note that counterclockwise rotations are considered positive and clockwise rotations are considered negative.) There is only one possible location of Q, as shown below.



Point Q could be also be described by different pairs of coordinates. For example, consider the coordinates $(2, 7\pi/3)$. Since $7\pi/3 = \pi/3 + 2\pi$, it follows that $(2, 7\pi/3)$ and $(2, \pi/3)$ are two different names for point Q. It is also possible to describe point Q using negative angles such as $(2, -5\pi/3)$ and $(2, -11\pi/3)$.

Real number r can be a negative number. For example, consider the polar coordinates (-2, $4\pi/3$). It is often easier to graph in the polar system by starting with the second number, and so begin with the ray which forms the angle $4\pi/3$ with the polar axis. This ray is drawn below with a solid line.



All r values on the this solid ray are considered positive. Negative r values are located on the *backward extension* of the ray, as shown by the dotted line. Therefore (-2, $4\pi/3$) is another name for point Q(2, $\pi/3$).

To summarize, we have discovered that point Q can be named by either of the following forms, where k represents any integer.

(2, $\pi/3 + k\,2\pi$)

(-2, $4\pi/3 + k\,2\pi$) Note that this could also be expressed as (-2, $\pi/3 + (2k + 1)\pi$)

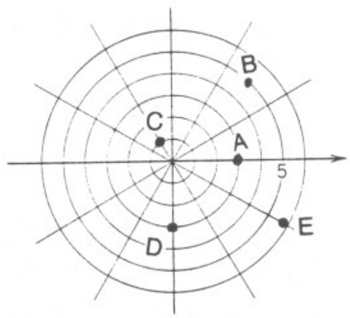
This could be generalized for every point in the polar coordinate system. Any point P(r, θ) can be expressed in either of the following forms, where k represents any integer.

(r, $\theta + k\,2\pi$)

(-r, $\theta + (2k + 1)\pi$)

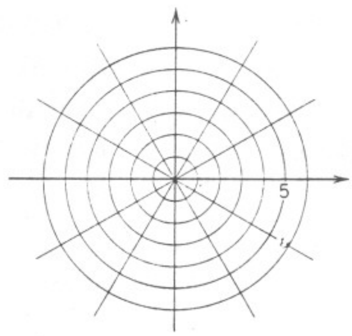
Problems

1. Give three different names for each of the points graphed on the right.



2. Locate each of the following points on the polar coordinate system given below.

P(3, $\pi/2$)	T(3, $-2\pi/3$)
Q(-6, π)	U(5, 11π)
R(4, $-\pi/6$)	V(5, $-3\pi/4$)
S(0, $\pi/8$)	W(-4, $25\pi/3$)



Answers

1. Point A: $(3, 0)$ $(-3, \pi)$ $(3, 2\pi)$

Point B: $(5, \pi/4)$ $(-5, 5\pi/4)$ $(-5, -3\pi/4)$

Point C: $(1, 2\pi/3)$ $(-1, -\pi/3)$ $(1, 8\pi/3)$

Point D: $(3, 3\pi/2)$ $(-3, \pi/2)$ $(3, 7\pi/2)$

Point E: $(-5.75, 5\pi/6)$ $(5.75, -\pi/6)$ $(-5.75, -7\pi/6)$

2.