PERMUTATIONS

A permutation is one of a collection of objects where order is important. For example, suppose you have a set of names and the first name drawn will win \$1000 and the second name drawn will win \$1. In this case it makes a significant difference whether a particular name is selected first or second.

Introductory problem

Suppose we have a five person committee. We wish to select two names -- the first person selected will be the chairperson and the second will be the treasurer. How many diffferent ordered pairs can we form?

If the designate the committee members by the letters A, B, C, D, and E, we can list all possible choices.

AB	BA
AC	CA
AD	DA
AE	EA
BC	CB
BD	DB
BE	EB
CD	DC
CE	EC
DE	ED

By counting the number of pairs, we see that there is a total of 20 different permutations.

Notation

Several notations are used to express the idea of n objects taken r at a time, were order is important. All of the following mean the same thing. ${}_nP_r$ $\quad P(n,r)$ $\quad P^n_r$

Computation

You can either use a calculator or the following formula: ${}_nP_r = \dfrac{n!}{(n-r)!}$.

We can use this formula on the "Introductory Problem" as follows:

$$ {}_5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5\times4\times3!}{3!} = 5\times4 = 20 $$

Harder Problem

Given a deck of 52 cards, how many ordered 5-card hands can be dealt?

$$ {}_{52}P_5 = \frac{52!}{(52-5)!} = \frac{52!}{47!} = \frac{52\times51\times50\times49\times48\times47!}{47!} = 52\times51\times50\times49\times48 = 311{,}875{,}200 $$