Factoring

Worked Examples

Factor completely: $y^5 - 6y^4 + 9y^3$

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Now you should recognize what remains as a perfect square:

$$y^5 - 6y^4 + 9y^3 = y^3\left(y^2 - 6y + 9\right) = y^3(y-3)^2$$

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But what if you can't find those two numbers? Or what if the factorization by grouping is too tricky? You can use the quadratic formula to find the same factors:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(8)(-7.5)}}{2(8)} = \frac{11 \pm \sqrt{361}}{16} = \frac{11 \pm 19}{16}.$$

The two roots are $x = \frac{15}{8}$ and $x = -\frac{1}{2}$. So this quadratic has $\left(x - \frac{15}{8}\right)$ and $\left(x + \frac{1}{2}\right)$ as factors. Multiplying by 8 so the leading coefficient is right, we get: $8x^2 - 11x - 7\frac{1}{2} = 8\left(x - \frac{15}{8}\right)\left(x + \frac{1}{2}\right)$.

(You should confirm that this is the same answer that we got from the AC method.)

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We can factor by guess and check. We need two numbers that add to –11 and multiply to make 24. –8 and –3 work. So

$$y^3 - 11y^2x + 24yx^2 = y\left(y^2 - 11yx + 24x^2\right) = y(y - 3x)(y - 8x).$$

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$$a^4 - 5a^2 - 36 = \left(a^2 - 9\right)\left(a^2 + 4\right)$$

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Now the factors really are quadratics that we recognize. The first is a difference of squares, and the second is a sum of squares (which is already completely factored).

$$a^4 - 5a^2 - 36 = \left(a^2 - 9\right)\left(a^2 + 4\right) = \left(a + 3\right)\left(a - 3\right)\left(a^2 + 4\right)$$