

Factoring (in particular, factoring quadratic trinomials)

Why do we need to be able to do this?

There are two main reasons to factor.

The first is to get more practice with multiplication. Factoring is using the inverse of multiplication, going backwards through the process. One reason to factor is to get a better and deeper understanding of multiplication. This deeper understanding will be helpful as we tackle more complicated algebra.

The second, more immediately practical reason, is to solve equations. The factors for a polynomial expression $P(x)$ give us the solutions to the equation $P(x) = 0$. The solutions are called zeros, or roots.

What should you be able to do?

Factor out common monomial factors.

Example: $70b^3 + 49b^6 = 7b^3(10 + 7b^3)$

Recognize and be able to factor a sum or difference of squares and perfect square trinomials. These “special products” come up all the time, and you should be able to handle them automatically. The sum and difference of cubes and perfect cubes come up less frequently.

Factor quadratic trinomials with leading coefficient of 1 – if they’re easy! This is a guess-and-check process; look for two numbers whose sum is the coefficient of the linear term (exponent = 1) and whose product is the constant term.

Example: Factor completely: $x^2 - 11x + 30$

I’m looking for two numbers who add to -11 and multiply to 30 ; -5 and -6 work. So the factorization is $x^2 - 11x + 30 = (x - 5)(x - 6)$.

Unfortunately, factoring this way can take a long time, and it’s hard to know if you should stop. If you can’t find a factorization, is it because you didn’t try the right factors yet, or maybe the factors involve square roots, or maybe the quadratic is already fully factored?

I usually don’t spend very long searching for a factorization of a quadratic trinomial. If I can’t see a factorization quickly, I turn to the quadratic formula (see below).

Use the quadratic formula to give you the roots, and use them to construct the factors. We usually use the connection between roots and factors (often called the Zero Product Property) to find roots from factors. But it goes the other way too. If you have the roots, you can use them to find the factors.

This method takes a little bit of time, too, but it will always give you an answer. This is also the easiest way to find factors that involve square roots.

Example: Factor completely: $10x^2 + 14x + 4$.

I don't immediately see a factorization, but I can use the quadratic formula to find the roots of $10x^2 + 14x + 4 = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-14 \pm \sqrt{(14)^2 - 4(10)(4)}}{2(10)} = \frac{-14 \pm \sqrt{196 - 160}}{20} = \frac{-14 \pm 6}{20}.$$

The two roots of the equation are -1 and $-\frac{2}{5}$, so the factors of $10x^2 + 14x + 4$ are $(x + 1)$

and $\left(x + \frac{2}{5}\right)$. I'll need to multiply by 10 so that the leading coefficient is right:

$$10x^2 + 14x + 4 = 10(x + 1)\left(x + \frac{2}{5}\right).$$

AC method: You may occasionally need to factor quadratic trinomials with leading coefficients different from 1. You can always use the quadratic formula to find the roots and construct the factors that way. Or you can use the "AC method," a sort of trick for factoring.

The trinomial $Ax^2 + Bx + C$ will factor if you can find two numbers whose sum is B and whose product is AC . Then you factor by grouping. You can use a 4-cell grid to keep track of the factors.

Example: Use the AC method to factor $10x^2 + 14x + 4$.

First, we need to factor out the common factor of 2. (But we'll have to remember it at the end of the problem!)

Now my problem is to factor $5x^2 + 7x + 2$. First, place the two outside terms into the two corner boxes.

$5x^2$	
	2

I want to split the middle term, the $7x$, across the two other cells, in such a way that the product of the two pieces is $10x^2$ (the AC). Now it's guess and check time: $3x$ and $4x$? No. $5x$ and $2x$? Yes!

$5x^2$	$5x$
$2x$	2

Now the last step is to factor by grouping. Pull out the common factors in the rows and columns:

	x	+	1
$5x$	$5x^2$		$5x$
+			
2	$2x$		2

The factorization is $10x^2 + 14x + 4 = 2(x+1)(5x+2)$. Is this the same answer we got when we used the quadratic formula before?