Exponents

Why do we need to be able to do this?

We need exponents to express very large or very small numbers (computers and calculators do, too). Exponents are part of the formulas that can tell us how much interest we'll earn, how long the roast needs to cook, how old the canvas is under the "antique" painting, the correct dosage for medicine, and how long it takes the well-hit baseball to leave the stadium.

What should you be able to do?

Rewrite numbers and algebraic expressions using exponents, including negative and fractional exponents. The first time we see exponents, they're introduced simply as shorthand notation for repeated multiplication. For example, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. This is handy, but it's not the only thing exponents can do. We can also use exponents to represent fractions, with the rule that $a^{-n} = \dfrac{1}{a^n}$. We can also use exponents to represent roots, with the rule that $a^{1/n} = \sqrt[n]{a}$.

Examples: $2^{10} = 1024$

$$2\frac{a}{b^3} = 2ab^{-3}$$

$$\sqrt{42x^3} = \left(42x^3\right)^{1/2}$$

(Note that all variables should represent positive real numbers in order to be sure that all these are correct.)

Use the rules of exponents to rewrite (simplify) expressions involving exponents. The rules of exponents are

- Rule of Exponents 1: $a^n \cdot a^m = a^{n+m}$. In English: When you multiply two powers with the same base, the exponents add.
- Rule of Exponents 2: $\dfrac{a^n}{a^m} = a^{n-m}$. In English: When you divide two powers with the same base, the exponents subtract.
- Rule of Exponents 3: $\left(a^m\right)^n = a^{mn}$. In English: When you raise a power to a power, the exponents multiply.
- $(ab)^n = a^n \cdot b^n$ and $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$. In English: When you multiply (or divide) two different powers, the exponent distributes.

- $a^{-n} = \dfrac{1}{a^n}$. In English: negative exponents represent reciprocals; a negative power means "one over." Notice that this is undefined if $a = 0$.
- $a^{1/n} = \sqrt[n]{a}$ In English: fractional exponents represent roots. Notice that this is not a real number if a is negative and n is even.
- For every nonzero base, $a^0 = 1$ and $a^1 = a$.

These rules can be used in either direction and in combination, depending on what you want your answer to look like.

Example:
$\dfrac{3^2 x^2}{3x^4} = \dfrac{3^2}{3} \cdot \dfrac{x^2}{x^4} = 3^1 x^{-2} = \dfrac{3}{x^2}$, provided x represents a nonzero number

Example:

$$2^{-1/2} = \left(2^{-1}\right)^{1/2} = \sqrt{\frac{1}{2}}, \textit{ or}$$

$$2^{-1/2} = \left(2^{1/2}\right)^{-1} = \frac{1}{\sqrt{2}}.$$

These are equivalent, but neither is the form your teacher might look for. The "simplified" form is $\dfrac{\sqrt{2}}{2}$.

Use your calculator to approximate powers, including roots. You can use your calculator to calculate any root by rewriting it as a power with a fractional exponent.

Example: $\sqrt[3]{35} = (35)^{1/3} \cong 3.27$.

(On an algebraic calculator, like the TI-84, remember to enclose the fractional exponent in parentheses.)