Equations of Lines

Worked Examples

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You could use the other point instead, or you could do some algebra to put this line in slope-intercept form – the equations would look different, but they would all represent the same line.

A faucet is dripping water at a constant rate into a bowl. At 1:00, there was ½ cup of water in the bowl. At 1:45, there was ¾ cup of water in the bowl.

How much water will be in the bowl at 3:30?

This is linear growth, because the faucet is dripping at a constant rate.

Let t be the time, measured in hours past noon, and let W be the amount of water in the bowl, measured in cups.

There are two points given: when $t = 1$, $W = 0.5$, and when $t = 1.75$, $W = 0.75$.

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The slope is rise/run, $\Delta W/\Delta t = \dfrac{(0.75 - 0.5)}{(1.75 - 1)} = \dfrac{.25}{.75} = \dfrac{1}{3}$ cups/hour.

So the equation will be: $W = \dfrac{1}{3}t + b.$

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To find the *W*-intercept, just plug in one of the points you know and solve for *b*:

$$\frac{1}{2} = \frac{1}{3} \cdot 1 + b, \ or \ b = \frac{1}{6}.$$

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As a check, let's make sure this gives us the right answer at the other known point – if I plug in $t = 1.75$, I get $W = 0.75$, which is right.

Finally, we're ready to answer the question. At 3:30, $t = 3.5$, and $W = 4/3$ cup.

So at 3:30, there will be 4/3 cups of water in the bowl.

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The slope of this line is also 2, so the lines have the same slope and they are parallel. (They are not the same line, because the two *y*-intercepts are different.)

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We'll let *t* be the number of minutes, and *C* will be the monthly cost (in dollars).

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The slope is the rate of change, so the slope is .15.

The flat fee each month represents the *y*-intercept, how much you pay if you make no calls, or the *C* value if *t* = 0. The *y*-intercept is 10.

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Now we can use the equation to find the cost when *t* = 45.

If you make 45 minutes of calls, the telephone plan will cost $16.75.