CONIC SECTIONS

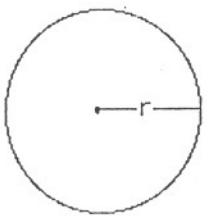
General comments

The conic sections are two-dimensional (flat) figures. Therefore, assume that all descriptions are for a plane.

In each of the following, the figure is in standard position. To obtain the general equation, replace x by x-h and replace y by y-k.

Circle

Definition: A circle consists of all points that are a given distance from some fixed point .



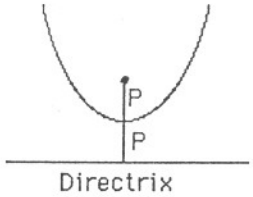
Equation: $x^2 + y^2 = r^2$

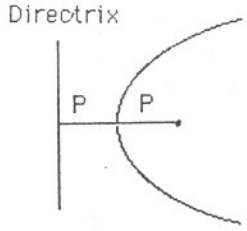
Note: A circle is a special case of an ellipse where a = b = r.

center at (0,0)
r = radius
eccentricity = e = 0

Parabola

Definition: A parabola is the set of all points that are equidistant from a fixed line (directrix) and a fixed point (focus) not on the line.



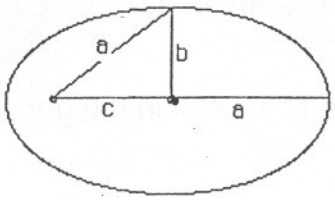


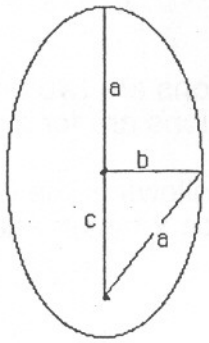
Equations: $x^2 = 4py$ (opens up) $x^2 = -4py$ (opens down) $y^2 = 4px$ (opens right) $y^2 = -4px$ (opens left)

vertex at (0,0)
p = distance between focus and vertex = distance between vertex and directrix

Ellipse

Definition: An ellipse is the set of all points the sum of whose distances from two distinct fixed points (foci) is a constant.





Equations: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

center at (0,0)

a = 1/2 major axis

b = 1/2 minor axis

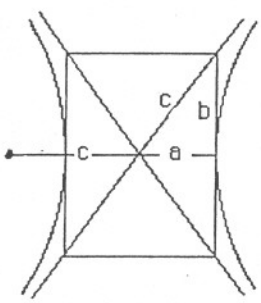
c = distance from center to focus

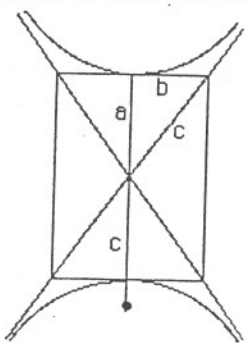
eccentricity = e = c/a and $0 < e < 1$

$c^2 = a^2 - b^2$

Hyperbola

Definition: A hyperbola is the set of all points the difference of whose distances from two distinct fixed points (foci) is constant.





Equations: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

center at (0,0)

c = distance between center and focus

a = distance between center and vertex

transverse axis = 2a

eccentricity = e = c/a and $e > 1$

$b^2 = c^2 - a^2$

Classifying a conic from its general equation

The graph of $Ax^2 + Cy^2 + Dx + Ey + F = 0$ is one of the following (except in degenerate cases).

If A = C, then it is a circle. (Note that a circle can be considered as a special ellipse.)

If A = 0 or C = 0 (but not both), then it is a parabola.

If A and C have like signs, then it is an ellipse.

If A and C have opposite signs, then it is a hyperbola.