

Arithmetic with Fractions

Worked Examples

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First, let's find a common denominator – in this case, just multiplying the denominators will give us the lowest common denominator, 24.

Then rewrite each fraction over that common denominator of 24.
(Yes, 3 is a fraction – it's the same as 3/1.)

$$3 + \frac{7}{8} + \frac{1}{3} = \frac{3}{1} \cdot \frac{24}{24} + \frac{7}{8} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{8}{8} = \frac{72}{24} + \frac{21}{24} + \frac{8}{24}$$

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Now we can add – combine the numerators over the common denominator:

$$3 + \frac{7}{8} + \frac{1}{3} = \frac{72}{24} + \frac{21}{24} + \frac{8}{24} = \frac{72 + 21 + 8}{24} = \frac{101}{24} = 4\frac{5}{24}$$

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This is the correct answer, but it isn't in simplest form.

$$6 \div \frac{4}{5} = \frac{30}{4} = \frac{15}{2} \cdot \frac{2}{2} = \frac{15}{2} \cdot 1 = \frac{15}{2} = 7\frac{1}{2}.$$

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$1/5$ is close to $1/4$.

$4/7$ is a little more than $1/2$.

$9/13$ is about $3/4$.

I estimate that this sum will be just about $1/4 + 1/2 + 3/4 + 3/4 = 2 \frac{1}{4}$

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Now let's work it out by hand: First we need a common denominator.

5, 7, 4, and 13 have no common factors (here's where knowing prime numbers is helpful). So the least common denominator is $5 \times 7 \times 4 \times 13 = 1820$.

$$1/5 = 1/5(364/364) = 364/1820$$

$$4/7 = 4/7(260/260) = 1040/1820$$

$$3/4 = 3/4(455/455) = 1365/1820$$

$$9/13 = 9/13(140/140) = 1260/1820$$

The sum, then, is $(364 + 1040 + 1365 + 1260)/1820 = 4029/1820$.

$4029 \div 1820 = 2$ with 389 remainder, so $4029/1820 = 2 \frac{389}{1820}$, which is in simplest form.

$$1/5 + 4/7 + 3/4 + 9/13 = ?$$

The sum is $4029/1820$ or $2 \frac{389}{1820}$, in simplest form.

How did our estimate do? We were pretty close – we estimated the sum would be about $2 \frac{1}{4}$, which is $2 \frac{455}{1820}$.

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Reality check: There's no way anyone will ask you to do this problem without your calculator. I used my calculator many times during the "by hand" part – to find the common denominator, to find the factors to multiply by, to do the addition, and to do the division.

How would you really do this problem? You'd reach for your calculator. Go ahead and find the sum $1/5 + 4/7 + 3/4 + 9/13$ on your calculator; I'll wait.

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Your calculator should have told you the sum was about 2.2137. (Which is, by the way, the same decimal the calculator gives for $4029/1820$.) If you didn't get this answer, then you should redo the calculation, carefully.

$$1/5 + 4/7 + 3/4 + 9/13 = ?$$

What do we still need to do by hand?

- Use our estimate (yes, 2.2137 is satisfyingly close to 2.25, so we punched things in correctly).
- If we need to have the answer expressed as a fraction, we can use our common denominator to find the fraction: Multiply the decimal number in your calculator by the common denominator, and it will show you the numerator of our improper fraction: 4029. Or you can multiply the part just to the right of the decimal to find the numerator of the fraction in the mixed number -- .2137 times 1820 is about 389, so our answer is $2 \frac{389}{1820}$.

Warning: Your calculator is a great tool, but it can't always tell you exactly the right answer. Often you will get an answer that is just a little bit off (that is, a little bit wrong). That's another reason you need to be able to think about these things yourself.