# Review of Integral Calculus Answers

1. The definite integral $\int_a^b f(x)\,dx$ is defined as $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$ with $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$. Geometrically, this gives the net area between $f(x)$ and the $x$-axis from $x = a$ to $x = b$.

2. The FTC has 2 parts.

   - Given $g(x) = \int_a^x f(t)\,dt$ for a continuous function $f(x)$ and $x \geq a$, we have that $g'(x) = f(x)$. (Another way to state this is that $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.)
   - For a continuous function $f(x)$, ${}_a^b f(x)\,dx = F(b) - F(a)$, where $F(x)$ is any antiderivative of $f(x)$.

3. (a) $\frac{d}{dx}[\int_3^x e^{t^2}\,dt\,] = e^{x^2}$ (See FTC part 1)

   (b) $\frac{d}{dx}[\int_x^3 e^{t^2}\,dt\,] = \frac{d}{dx}[-\int_3^x e^{t^2}\,dt\,] = -e^{x^2}$

   (c) $\frac{d}{dx}[\int_0^{x^3} e^{t^2}\,dt\,] = e^{(x^3)^2}\cdot 3x^2 = 3x^2e^{x^6}$ (Using FTC part 1 and the chain rule)

   (d) $\frac{d}{dx}[\int_0^4 e^{t^2}\,dt\,] = 0$ ($\int_0^4 e^{t^2}\,dt$ has a constant value)

4. Note: Many of these integrals can be evaluated using different methods. I have outlined one way to integrate.

   (a) $\int_0^1 \frac{x}{4-x^2}\,dx = \frac{1}{2}\ln(\frac{4}{3})$ (Using substitution $u = 4 - x^2, \quad du = -2x\ dx$)

   (b) $\int_0^1 \frac{1}{4-x^2}\,dx = \frac{1}{4}\ln(3)$ (Using partial fraction decomposition)

   (c) $\int_0^1 \frac{1}{4+x^2}\,dx = \frac{1}{2}\arctan(\frac{1}{2})$ (Factoring 4 out of the denominator and using substitution $u = \frac{x}{2}$)

   (d) $\int_0^\infty xe^{-x^2}\,dx = \frac{1}{2}$ (Using limits since it is improper and using the substitution $u = -x^2$ to integrate)

   (e) $\int_0^\infty xe^{-x}\,dx = 1$ (Using limits since it is improper and using integration by parts to integrate)

(f) $\displaystyle\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx = \frac{\pi}{2}$ (The antiderivative of $\frac{1}{\sqrt{1-x^2}}$ is nice.....it's $\arcsin x$)

(g) $\displaystyle\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx = \ln(\sqrt{2}+1)$ (Using trig. substitution $x = \tan x$)

(h) $\displaystyle\int_0^1 (x+1)\sqrt{x}\,dx = \frac{16}{15}$ (Distribute the $\sqrt{x}$ and simplify)

(i) $\displaystyle\int_0^1 x\sqrt{x+1}\,dx = \frac{4}{15}(1+\sqrt{2})$ (Using a rationalizing subst. $u = \sqrt{x+1}$)

(j) $\displaystyle\int_{-1}^1 \frac{x}{1+x^8}\,dx = 0$ (Since $\frac{x}{1+x^8}$ is odd)

(k) $\displaystyle\int \frac{\cos(3x)}{1+4\sin(3x)}\,dx = \frac{1}{12}\ln|1+4\sin(3x)| + C$ (Using subst. $u = 1+4\sin(3x)$)

(l) $\displaystyle\int_0^\infty \frac{x}{1+x^4}\,dx = \frac{\pi}{4}$ (Using limits since it is improper and using the subst. $u = x^2$ to integrate)

(m) $\displaystyle\int_0^{\pi/2} \cos^3 x\,dx = \frac{2}{3}$ (Using the trig. identity $\cos^2 x = 1 - \sin^2 x$ and the subst. $u = \sin x$)

(n) $\displaystyle\int \arctan x\,dx = x\arctan x - \frac{1}{2}\ln(1+x^2) + C$ (Using integration by parts)