

Section 9.2: Polynomial Functions

Due: Tuesday 5/25 (with your homework) Try to take a look at this before Monday's lecture. It will make the lecture on §9.3 much easier.

A polynomial is a sum of power functions kx^p where p is a positive integer. (k can be any #.)

Examples: $f(x) = 5x^4 - 3x^3 + 2x - 9$ ← (The power on -9 is $p = 0$.), $g(x) = \sqrt{2}x^5 + x^2 - 2.8x^{10} - \frac{1}{2}x$

(A function like $h(x) = 2x^2 + x^{1/2}$ is **not** a polynomial because the power $p = \frac{1}{2}$ is not a positive integer.)

***Question 1:** Which of the following are polynomials?

i) $y = 2.35x^4 - \frac{3}{7}x^2 + 3$, ii) $y = 5x^2 - x + \frac{2}{x}$ iii) $y = \ln(3) \cdot x^{100} + 7x$

Definitions:

- Degree of a polynomial: highest power/exponent of all the terms
- Leading term: Term with the highest power/exponent
- Leading coefficient: Coefficient (k) of the leading term

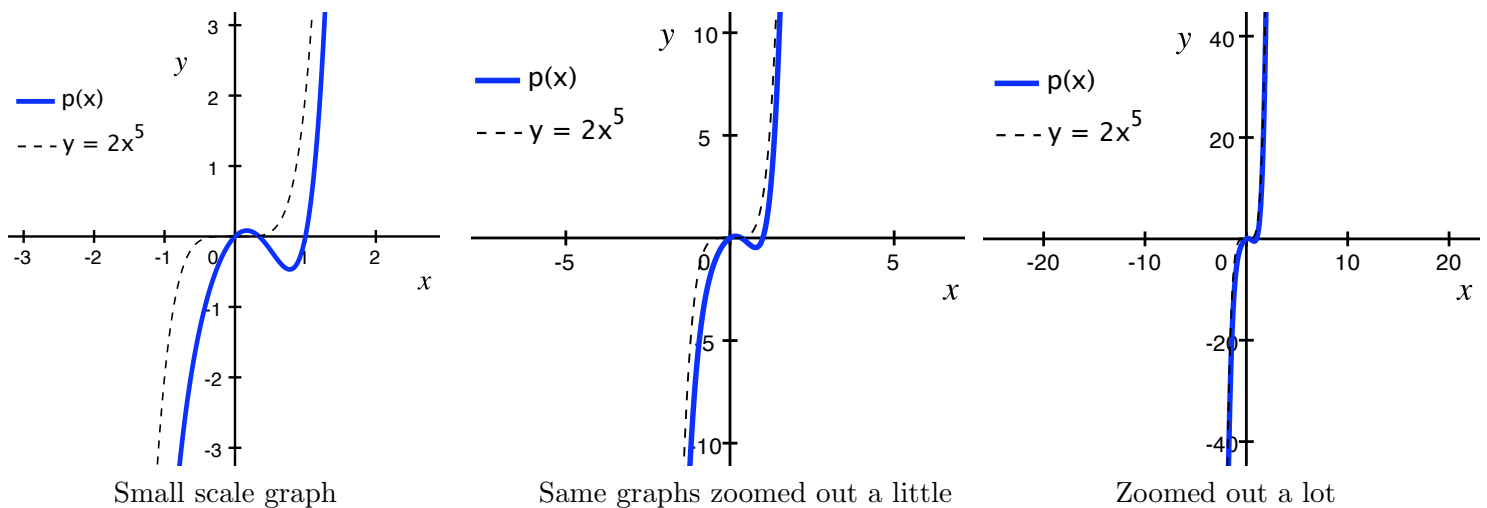
Example: The function $f(x) = 5x^4 - 3x^3 + 2x - 9$ has degree 4, a leading term of $5x^4$, and a leading coefficient of 5.

***Question 2:** What is the degree, leading term, and leading coefficient of $g(x) = 7x^8 + 5x^3 - 4x^2 + x + 1$?

***Question 3:** What is the degree, leading term, and leading coefficient of $g(x) = \sqrt{2}x^5 + x^2 - 2.8x^{10} - \frac{1}{2}x$?

Long-Run Behavior of Polynomials: "Long-Run" means that we are looking at what happens to the functions as $x \rightarrow \infty$ or $x \rightarrow -\infty$. (What happens as we look at the right and left portions of $f(x)$ beyond our graphs?)

Example: Consider $p(x) = 2x^5 - 3x^2 + x$. Below are graphs of $p(x)$ and $y = 2x^5$ using different scales.



When zoomed in close, you can see how different $p(x) = 2x^5 - 3x^2 + x$ and $y = 2x^5$ are, but as you zoom further and further out, they look almost the same!

So, as $x \rightarrow \infty$, $p(x) \rightarrow \infty$, which means it increases more and more. As $x \rightarrow -\infty$, we have that $p(x) \rightarrow -\infty$, since as x becomes more negative, the function also gets more negative.

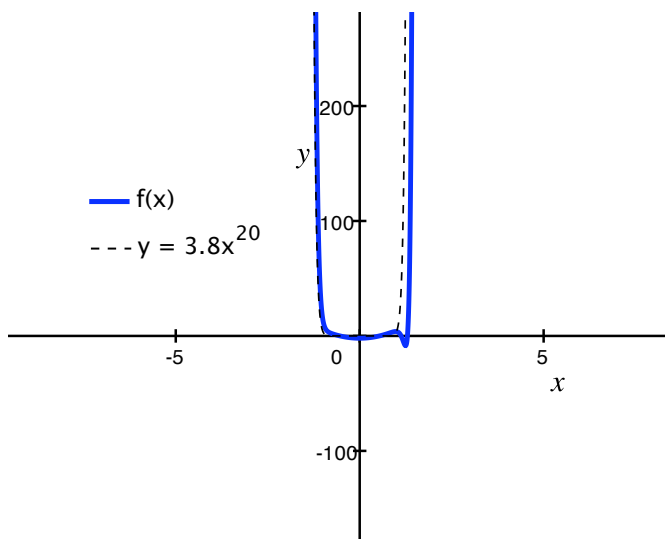
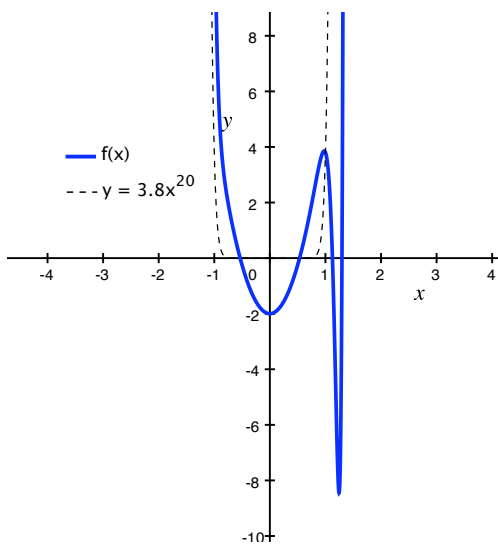
General Rule: If ax^n is the leading term of a polynomial function $p(x)$, then in the long-run (as $x \rightarrow \infty$ or as $x \rightarrow -\infty$), the polynomial acts like and looks like ax^n . This means that if you zoom out far enough, the polynomial will look like ax^n .

Question 4: In the long-run, what does $q(x) = 9x^4 - 3x^3 - 2x^2 + x - 3$ behave like? What does this mean for the shape of the graph? As $x \rightarrow \infty$, what happens to the function $q(x)$? As $x \rightarrow -\infty$, what happens to $q(x)$?

Question 5: In the long-run, what does $p(x) = -2x^3 + x^2 - 1.2x + 9$ behave like? What does this mean for the shape of the graph? As $x \rightarrow \infty$, what happens to the function $p(x)$? As $x \rightarrow -\infty$, what happens to $p(x)$?

Now, even with the grossest polynomials, you can picture their general shape!!

For example, $f(x) = 3.8x^{20} - 5x^{19} + 7x^2 - 2$ is going to have a shape similar to $y = 3.8x^{20}$, which is a U-shaped curve that opens upward. The function $f(x)$ has more wiggles than $y = 3.8x^{20}$ if you zoom in on the graphs, but at least we have an idea of what its overall shape will be. (See below.)



Exciting!!